38 JUNE 2016 OBJECTIVE TEST

7. Simplify: $\dfrac{3^{n-1} \times 27^{n+1}}{81^n}$.

A. 3^{2n} B. 9

C. 3^n D. 3^{n+1}

therefore he is a good footballer.

D. Joseph plays good football therefore he is a Nigerian footballer.

6. On a map, 1 cm represents 5 km. Find the area on the map that represents 100 km².

A. 2 cm²   B. 4 cm²

C. 8 cm²   D. 16 cm²

expression $\dfrac{y^2 - 9y + 10}{y^2 + 4y - 21}$ is undefined.

A. 6, –7   B. 3, –6

C. 3, –7   D. –3, –7

12. Given that $2x + y = 7$ and $3x - 2y = 3$, by how much is $7x$ greater than 10?

A. 1   B. 3

C. 7   D. 17

13. Simplify: $\dfrac{2}{1-x} - \dfrac{1}{x}$.

A. $\dfrac{x+1}{x(1-x)}$

B. $\dfrac{3x-1}{x(1-x)}$

C. $\dfrac{3x+1}{x(1-x)}$

D. $\dfrac{x-1}{x(1-x)}$

14. Make s the subject of the relation:

$p = s + \dfrac{sm^2}{nr}$.

A. $s = \dfrac{mrp}{nr+m^2}$

B. $s = \dfrac{nr+m^2}{mrp}$

C. $s = \dfrac{nrp}{mr+m^2}$

D. $s = \dfrac{nrp}{nr+m^2}$

15. Factorize: $(2x+3y)^2 - (x-4y)^2$
    A. $(3x-y)(x+7y)$
    B. $(3x+y)(2x-7y)$
    C. $(3x+y)(x-7y)$
    D. $(3x-y)(2x+7y)$

16. The curved surface area of a cylinder, 5 cm high, is 110 cm^2. Find the radius of its base. [Take $\pi = \dfrac{22}{7}$]
    A. 2.6 cm
    B. 3.5 cm
    C. 3.6 cm
    D. 7.0 cm

17. The volume of a pyramid with height 15 cm is 90 cm^3. If its base is a rectangle with dimensions x cm by 6 cm, find the value of x.
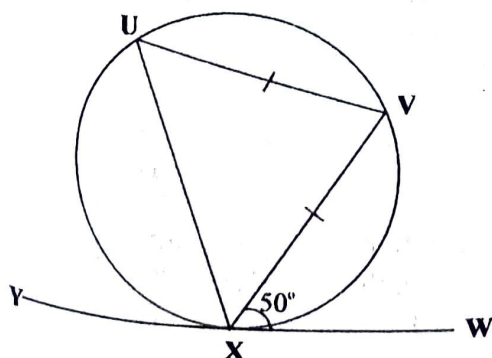   A. 3
    B. 5
    C. 6
    D. 8

18.

In the diagram $\overline{YW}$ is a tangent to the circle at X, $|UV| = |VX|$ and $\angle VXW = 50°$. Find the value $\angle UXY$.
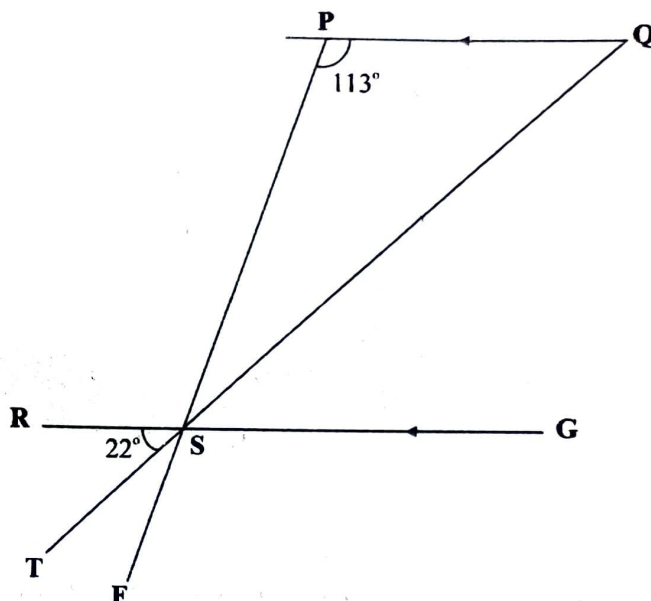A. 70°
B. 80°
C. 105°
D. 110°

19.

In the diagram, $\overline{PF}, \overline{QT}, \overline{RG}$ intersect at S and $PQ//RG$. If $\angle SPQ = 113°$ and $\angle RST = 22°$, find $\angle PSQ$.
A. 22°
B. 45°
C. 67°
D. 89°
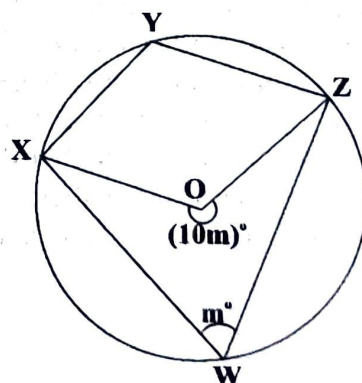
20.

In the diagram, O is the centre of the circle, $\angle XOZ = (10m)°$ and $\angle XWZ = m°$. Calculate the value of m.
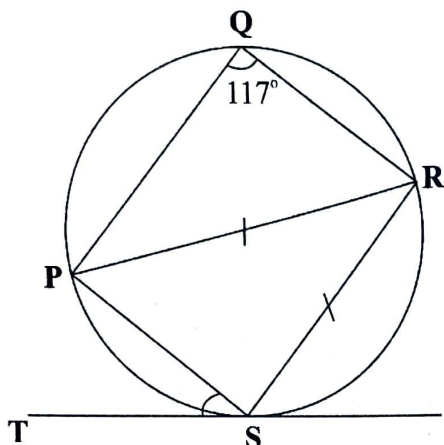A. 30
B. 36
C. 40
D. 72

21. Kweku walked 8 m up a slope and was 3 m above the ground. If he walks 12 m further up the slope, how far above the ground will he be?
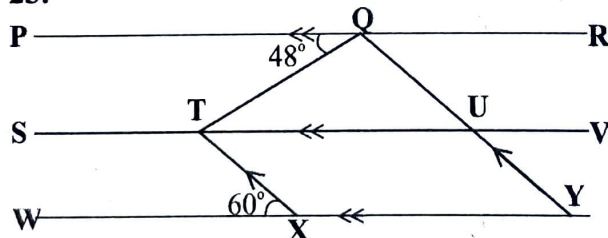
   A. 4.5 m   B. 6.0 m

   C. 7.5 m   D. 9.0 m

22.

In the diagram, *TS* is a tangent to the circle at *S*. $|PR| = |RS|$ and $\angle PQR = 117°$.

Calculate $\angle PST$.

   A. 54°   B. 44°

   C. 34°   D. 27°

23.

In the diagram *PR//SV//WY, TX//QY,* $\angle PQT = 48°$ and $\angle TXW = 60°$.

Find $\angle TQU$.

   A. 120°   B. 108°

   C. 72°   D. 60°

A straight line passes through the points *P*(1, 2) and *Q*(5, 8). *Use this information to answer questions **24** and **25**.*

24. Calculate the gradient of the line *PQ.*

   A. $\dfrac{3}{5}$ B. $\dfrac{2}{3}$

   C. $\dfrac{3}{2}$ D. $\dfrac{5}{3}$

25. Calculate the length *PQ.*

   A. $4\sqrt{11}$ B. $4\sqrt{10}$

   C. $2\sqrt{17}$ D. $2\sqrt{13}$

26.
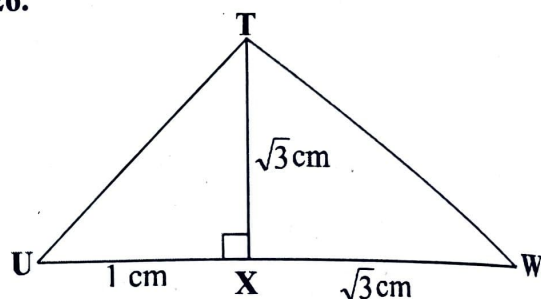
In the diagram, *TX* is perpendicular to *UW*, $|UX| = 1$ cm and $|TX| = |WX| = \sqrt{3}$ cm. Find $\angle UTW$.

   A. 135°   B. 105°

   C. 75°   D. 60°

27. If $\cos \theta = x$ and $\sin 60° = x + 0.5$, $0° \leq \theta \leq 90°$, find, **correct** to the **nearest** degree, the value of θ.

   A. 66°   B. 67°

   C. 68°   D. 69°

Age (Years)	13	14	15	16	17
Frequency	10	24	8	5	3

The table shows the ages of students in a club. *Use it to answer questions **28** and **29**.*

28. How many students are in the club?

   A. 50   B. 55

   C. 60   D. 65

29. Find the median age.
A. 13
B. 14
C. 15
D. 16

30.
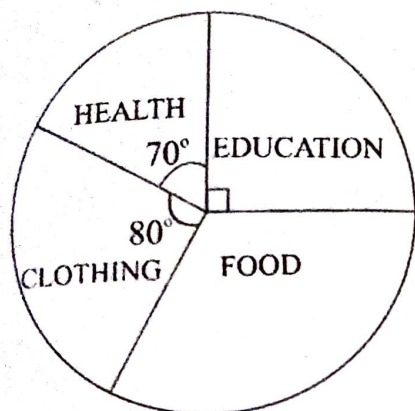
The figure is a pie chart which represents the expenditure of a family in a year. If the total income of the family was Le10, 800,000.00, how much was spent on food?
A. Le 2,250,000.00
B. Le 2,700,000.00
C. Le 3,600,000.00
D. Le 4,500,000.00

31. A fair die is thrown two times. What is the probability that the sum of the scores is at least 10?
A. $\dfrac{5}{36}$
B. $\dfrac{1}{6}$
C. $\dfrac{5}{18}$
D. $\dfrac{2}{3}$

32. The marks of eight students in a test are: 10, 4, 5, 3, 14, 13, 16 and 7. Find the range.
A. 16
B. 14
C. 13
D. 11

33. If $\log_2(3x - 1) = 5$, find x.
A. 2.00
B. 3.67
C. 8.67
D. 11.00

34. A sphere of radius r cm has the same volume as a cylinder of radius 3 cm and height 4 cm. Find the value of r.

A. $\dfrac{2}{3}$
B. 2
C. 3
D. 6

35. Express 1975 correct to 2 significant figures.
A. 20
B. 1900
C. 1980
D. 2000

36.
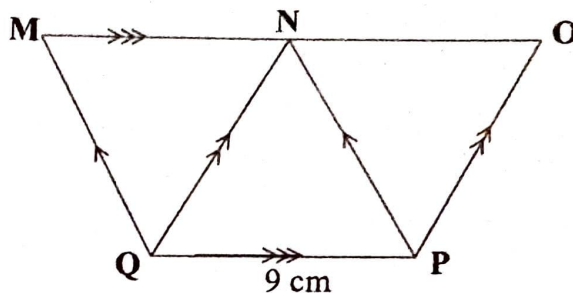
In the diagram *MOPQ* is a trapezium with *QP//MO*, *MQ//NP*, *NQ//OP*, /*QP*/ = 9 cm and the height of triangle *QNP* = 6 cm. Calculate the area of the trapezium.
A. 96 cm²
B. 90 cm²
C. 81 cm²
D. 27 cm²

37. The perimeter of a sector of a circle of radius 21 cm is 64 cm. Find the angle of the sector. [Take $\pi = \frac{22}{7}$]
A. 70°
B. 60°
C. 55°
D. 42°

38.
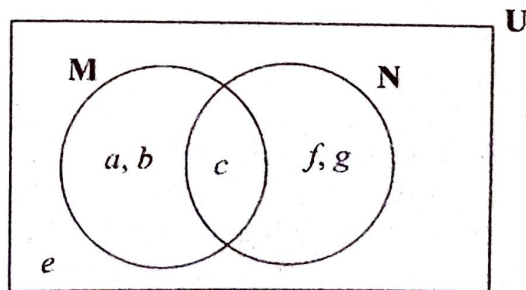
Determine $M \cap N$ from the Venn diagram.
A. {*f, g*}
B. {*e*}
C. {*c, f, g*}
D. {*e, f, g*}

39. If 20(mod 9) is equivalent to y(mod 6), find y.

A. 1
B. 2
C. 3
D. 4

40. Simplify: $\dfrac{(p-r)^2 - r^2}{2p^2 - 4pr}$.

A. $\dfrac{1}{2}$
B. $p - 2r$
C. $\dfrac{1}{p - 2r}$
D. $\dfrac{2p}{p - 2r}$

41.

In diagram, O is the centre of the circle, $\angle QPS = 100°$, $\angle PSQ = 60°$ and $\angle QSR = 80°$. Calculate $\angle SQR$.

A. 20°
B. 40°
C. 60°
D. 80°

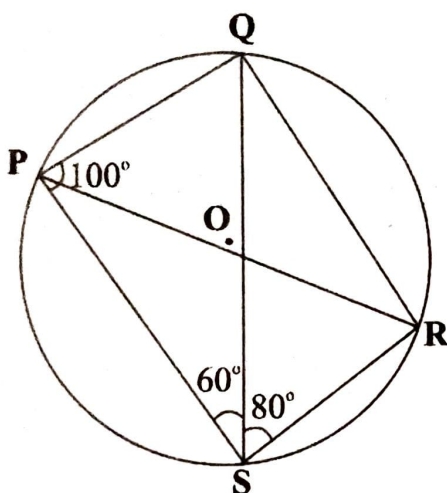**42.** A bag contains 5 red and 4 blue identical balls. If 2 balls are selected at random from the bag, one after the other, with replacement, find the probability that the first is red and the second blue.

A. $\dfrac{2}{9}$
B. $\dfrac{5}{18}$
C. $\dfrac{20}{81}$
D. $\dfrac{5}{9}$

43. The relation $y = x^2 + 2x + k$ passes through the point (2, 0). Find the value of k.

A. –8
B. –4
C. 4
D. 8

44. Find the next three terms of the sequence: 0, 1, 1, 2, 3, 5, 8,...

A. 13, 19, 23
B. 9, 11, 13
C. 11, 15, 19
D. 13, 21, 34

45. Find the lower quartile of the distribution illustrated by the cumulative frequency curve.

A. 17.5
B. 19.0
C. 27.5
D. 28.0
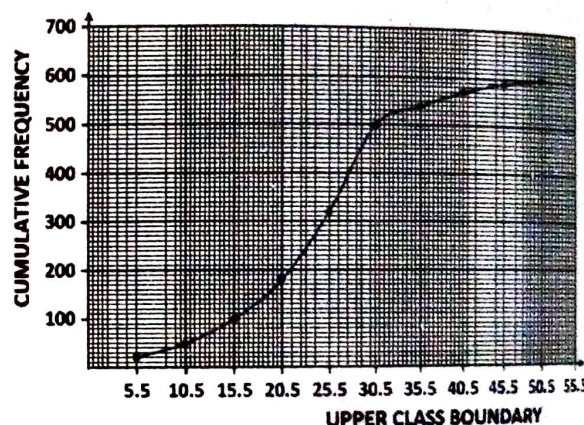
46. The ratio of the exterior angle to the interior angel of a regular polygon is 1:11. How many sides has the polygon?

A. 30
B. 24
C. 18
D. 12

47. Halima is n years old. Her brother's age is 5 years more than half of her age. How old is her brother?

A. $\dfrac{n}{2} + \dfrac{5}{2}$
B. $\dfrac{n}{2} - 5$
C. $5 - \dfrac{n}{2}$
D. $\dfrac{n}{2} + 5$

48.
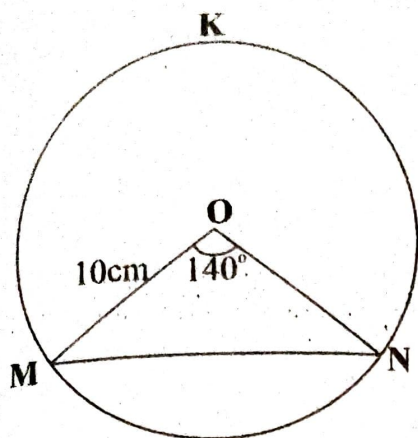
In the diagram $\overline{MN}$ is a chord of a circle KMN centre O and radius 10 cm.

If $\angle MON = 140°$, find, correct to the nearest cm, the length of the chord MN.

A. 19 cm   B. 18 cm

C. 17 cm   D. 12 cm

49. An object is 6 m away from the base of a mast. If the angle of depression of the object from the top of the mast is $50°$, find, correct to 2 decimal places, the height of the mast.

A. 8.60 m   B. 7.83 m

C. 7.51 m   D. 7.15 m

50.
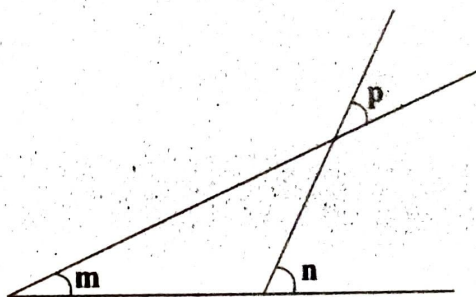
From the diagram, which of the following is **true**?

A. $m + n + p = 180°$

B. $m + n = 180°$

C. $m = p + n$

D. $n = m + p$

2016 THEORY QUESTIONS AND ANSWERS

Question 1

a) Simplify, without using mathematical tables or calculator, $\dfrac{4\frac{1}{4} - 3\frac{1}{2} + 3\frac{1}{8}}{3\frac{2}{5} \text{ of } 1\frac{1}{4} \div 2\frac{5}{6}}$.

b) If two numbers are selected at random, one after the other, with replacement from the set $A = \{5, 6, 7, 8, 9\}$, find the probability of selecting at least one prime number.

Solution

a) $\dfrac{4\frac{1}{4} - 3\frac{1}{2} + 3\frac{1}{8}}{3\frac{2}{5} \text{ of } 1\frac{1}{4} \div 2\frac{5}{6}}$

$= \dfrac{\frac{17}{4} - \frac{7}{4} + \frac{25}{8}}{\frac{17}{5} \times \frac{5}{4} \div \frac{17}{6}}$

$= \dfrac{\frac{34-28+25}{8}}{\frac{17}{5} \times \frac{5}{4} \times \frac{6}{17}}$

$= \dfrac{\frac{31}{8}}{\frac{3}{2}} = \dfrac{31}{8} \times \dfrac{2}{3} = \dfrac{31}{12} = 2\frac{7}{12}$

b)

	5	6	7	8	9
5	5, 5	5, 6	5, 7	5, 8	5, 9
6	6, 5	6, 6	6, 7	6, 8	6, 9
7	7, 5	7, 6	7, 7	7, 8	7, 9
8	8, 5	8, 6	8, 7	8, 8	8, 9
9	9, 5	9, 6	9, 7	9, 8	9, 9

Pairs containing at least one prime number
= {(5, 5), (5, 6), (5, 7), (5, 8), (5, 9), (6, 5), (6, 7), (7, 5), (7, 6), (7, 7), (7, 8), (7, 9), (8, 5), (8, 7), (9, 5), (9, 7)}

Probability of selecting at least one prime number

$= \dfrac{16}{25}$

Question 2

a) Given that $\cos x = \frac{3}{5}, 0° < x < 90°$, calculate, without using mathematical table or calculator $\dfrac{3 \tan x}{2 \sin x + 3 \cos x}$.

b)
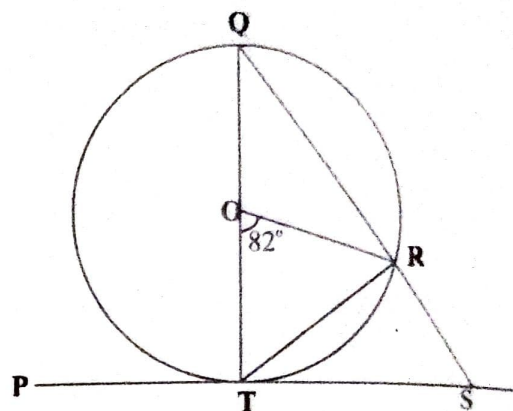
In the diagram, PS is tangent to the circle of centre O. If QS is a straight line and $\angle TOR = 82°$, find $\angle RST$.

Solution

a) $\cos x = \dfrac{3}{5}$
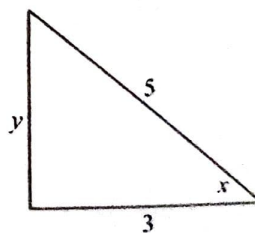
From the diagram,
$y = \sqrt{5^2 - 3^2} = 4$

$\Rightarrow \tan x = \dfrac{4}{3}$ and $\sin x = \dfrac{4}{5}$

$\dfrac{3 \tan x}{2 \sin x + 3 \cos x}$

$= \dfrac{3(\frac{4}{3})}{2(\frac{4}{5}) + 3(\frac{3}{5})}$

$= \dfrac{4}{\frac{17}{5}} = 4 \times \dfrac{5}{17} = \dfrac{20}{17} = 1\frac{3}{17}$

b) Triangle OTR is isosceles since $OT = OR$ (radii)

$$\angle ORT = \angle OTR = \frac{180° - 82°}{2} = 49°$$

$\Rightarrow \angle OTS = 90°$ (PT is a tangent at T)

$\Rightarrow \angle RTS = 90° - 49° = 41°$

$\Rightarrow \angle TOR = 2\angle TQR$

$\Rightarrow \angle TQR = \frac{1}{2}\angle TOR = \frac{1}{2} \times 82° = 41°$

But $\angle TRS = \angle OTR + \angle OQR$

$$= 49° + 41° = 90°$$

Considering triangle TRS

$\angle RST + \angle TRS + \angle RTS = 180°$

$\Rightarrow \angle RTS = 180° - (90° + 41°) = 49°$

Question 3

a) For what values of x is the expression

$$\frac{5}{x^2 + 2x - 8}$$ **not** defined?

b) Three times the age of Felicia is four more than the age of Asare. In three years, the sum of their ages will be 30 years. Find their present ages.

Solution

a) For the expression not to be defined

$x^2 + 2x - 8 = 0$

$\Rightarrow x^2 + 4x - 2x - 8 = 0$

$\Rightarrow (x + 4)(x - 2) = 0$

$\Rightarrow x = -4$ or $x = 2$

b) Let Felicia's age be x and Asare's be y

$3x = 4 + y$ ....(1) and

$x + 3 + y + 3 = 30$ ....(2)

From (1) $y = 3x - 4$

Putting $y = 3x - 4$ into (2)

$x + 3 + 3x - 4 + 3 = 30$

$4x = 28$ $\Rightarrow x = 7$

$y = 3 \times 7 - 4 = 21 - 4 = 17$

$\therefore$ Felicia is 7 years and Asare is 17 years now.

Question 4

If P, Q and R are sets such that $n(P) = 20$, $n(Q) = 16$, $n(R) = 21$, $n(P \cap Q) = 7$, $n(P \cap R) = 8$, $n(Q \cap R) = 5$ and $n(P \cap Q \cap R) = 3$,

a) represent this information on a Venn diagram

b) find:
   i) $n(P \cup Q \cup R)$;
   ii) the probability of $((P \cup Q)' \cap R)$.

Solution

a) Let x, y and z represent only P, Q and R respectively i.e.

$((R \cup Q)' \cap P) = x$; $((P \cup R)' \cap Q) = y$

$((P \cup Q)' \cap R) = z$
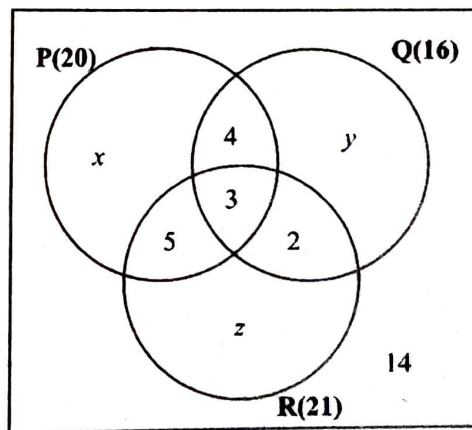
b) From the Venn diagram,

$x + 4 + 3 + 5 = 20 \Rightarrow x = 8$

$y + 4 + 3 + 2 = 16 \Rightarrow y = 7$

$z + 5 + 3 + 2 = 21 \Rightarrow z = 11$

i) $n(P \cup Q \cup R) = 8 + 4 + 3 + 5 + 7 + 2 + 11$

$\qquad\qquad = 40$

ii) $((P \cup Q)' \cap R) = z = 11$

Probability of $((P \cup Q)' \cap R) = \dfrac{11}{40}$

Question 5
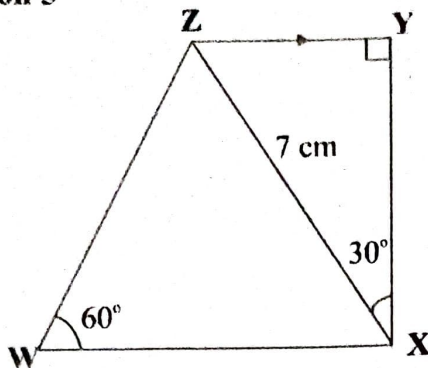
In the diagram *WXYZ* is a trapezium, $|XZ| = 7$cm, $\angle ZYX = 90°$, $\angle ZWX = 60°$ and $\angle ZXY = 30°$. Calculate, correct to the nearest whole number, the area of *WXYZ*.

Solution
Considering triangle *ZYX*

$$\sin 30° = \frac{|ZY|}{7} \implies |ZY| = 7\sin 30° = 3.5$$

$$\cos 30° = \frac{|YX|}{7} \implies |ZY| = 7\cos 30° = 6.06$$

Triangle *WZX* is equilateral triangle since
$\angle ZWX = 60°$
$\implies |WU| = |UX|$
From the diagram
$$\cos 60° = \frac{|WM|}{7} \implies |ZY| = 7\cos 60° = 3.5$$
$$\implies |WX| = 3.5 + 3.5 = 7 \text{ cm}$$
Area of *WXYZ* = Area of trapezium
$$= \frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$$

Area of *WXYZ*
$$= \frac{1}{2}(3.5 + 7) \times 6.06 = 31.815 = 32 \text{ cm}^2$$

Question 6
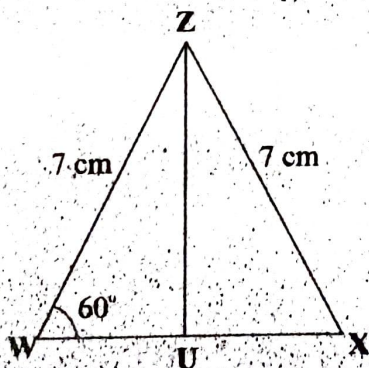a) If *p* varies directly as t^3 and $p = 9.6$ when $t = 4$, find *t* when $p = 150$.

b) A farmer has 1 hectare of land. One half of the land was used for planting oranges, $\frac{1}{3}$ of the remainder was used for planting mangoes while plantain was planted on the rest.
i) Express the area of land used for mangoes as a fraction of that used for plantain.

ii) If a labourer was given a week to weed the orange plantation and he completes $\frac{1}{5}$ of it on the first day, what area, in square metres, was left?
[Take 1 hectare = 10,000 m²]

Solution
$p \propto t^3 \implies p = kt^3$, where *k* is a constant
When $p = 9.6$, $t = 4$
$$\implies 9.6 = k(4)^3 \implies k = \frac{9.6}{64} = 0.15$$
Substituting
$$\implies p = 0.15t^3$$
When $p = 150$
$$\implies 150 = 0.15t^3 \implies t^3 = 1000 \implies t = 10$$

b) Let x = the land
$\implies$ Orange land $= \frac{1}{2}x$
Remainder $= x - \frac{1}{2}x = \frac{1}{2}x$
Mango land $= \frac{1}{3} \times \frac{1}{2}x = \frac{1}{6}x$
Remaining land $= \frac{1}{2}x - \frac{1}{6}x = \frac{1}{3}x$
$\implies$ Plantain land $= \frac{1}{3}x$

i) $\dfrac{\text{Land used for mangoes}}{\text{Land used for plantain}}$

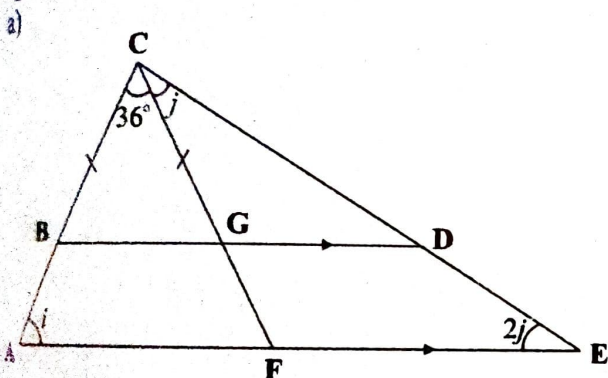$= \dfrac{\frac{x}{6}}{\frac{x}{3}} = \dfrac{x}{6} \times \dfrac{3}{x} = \dfrac{1}{2}$

ii) $\dfrac{1}{5}$ of orange plantation $= \dfrac{1}{5} \times \dfrac{1}{2} x = \dfrac{x}{10}$

Area of land left

$= \dfrac{1}{2} x - \dfrac{1}{10} x = \dfrac{2}{3} x$

$= \dfrac{2}{3} \times 10000 = 4{,}000 \text{ m}^2$

Question 7

a)

In the diagram, ACE is a triangle, CF is a straight line, $BD//AE$ and $|BC| = |CG|$.

If $\angle BCG = 36°$, $\angle BAF = i$, $\angle GCD = j$ and $\angle DEF = 2j$, find the values of i and j.

b) An aeroplane flies from P to Q in 1 hour at a speed of 120 km/min., where P and Q are on the parallel of latitude $60°\ N$. If the aeroplane flies along this parallel of latitude, calculate, correct to **three** significant figures, the difference in longitudes of P and Q.

[Take $\pi = \frac{22}{7}$, radius of the earth $= 6400$ km]

Solution

Triangle CBG is isosceles triangle

$36° + 2\angle CBG = 180°$

$\Rightarrow \angle CBG = \dfrac{180° - 36°}{2} = 72°$

$BD//AE \quad \Rightarrow \quad \angle CBG = \angle BAF = 72°$

$\Rightarrow i = 72°$

$\angle CFE = \angle ACF + \angle CAF$

(Exterior angle of a triangle is equal to the sum of interior opposite angles

$\Rightarrow \angle CFE = 72° + 36° = 108°$

Considering triangle CFE

$\Rightarrow j + 108° + 2j = 180°$

$\Rightarrow j = \dfrac{72°}{3} = 24°$

b) Distance travelled along the parallel of latitudes $PQ = \dfrac{\theta}{360} \times 2\pi r$

But $r = R\cos 60° = 6400 \cos 60 = 3200$

$\Rightarrow |PQ| = \dfrac{\theta}{360} \times 2\pi \times 3200$

This distance travelled at a speed of 120 km/min in 1 hour (60 minutes)

Distance travelled

$= \text{speed} \times \text{time}$

$= 120 \text{ km/min} \times 60 \text{ min} = 7200 \text{ km}$

$\therefore \quad 7200 = \dfrac{\theta}{360} \times 2\pi \times 3200$

$\Rightarrow \theta = \dfrac{7200 \times 360}{2\pi \times 3200} = 128.9 = 129° \ (3\text{s.g})$

Question 8

a) Using ruler and a pair of compasses only, construct quadrilateral, $PQRS$, such that $|PQ| = 8$ cm, $|SQ| = 10.2$cm, $|QR| = 7.5$cm, $\angle QPS = 75°$ and $PS // QR$.

b) i) Draw locus, l_1, of points equidistant from SR and QR;

   ii) Draw locus, l_2 of points equidistant from P and Q;

c) Measure $|TQ|$, where T is the point of intersection of l_1 and l_2.

Solution

$|TQ| = 5.2$ cm

Question 9

a) Copy and complete the table of values for the relation $y = x^2 - 5x + 5$ for $-1 \leq x \leq 6$.

x	-1	0	1	2	3	4	5	6
y		5	1				5	

b) Using scales of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 2 units on the y-axis, draw the graph of $y = x^2 - 5x + 5$ for $-1 \leq x \leq 6$.

c) Use the graph to find the:
   i) minimum value of y;
   ii) roots of $x^2 - 5x + 5 = 0$;
   iii) solution of $x^2 + 2x + 5 = 7x + 2$.
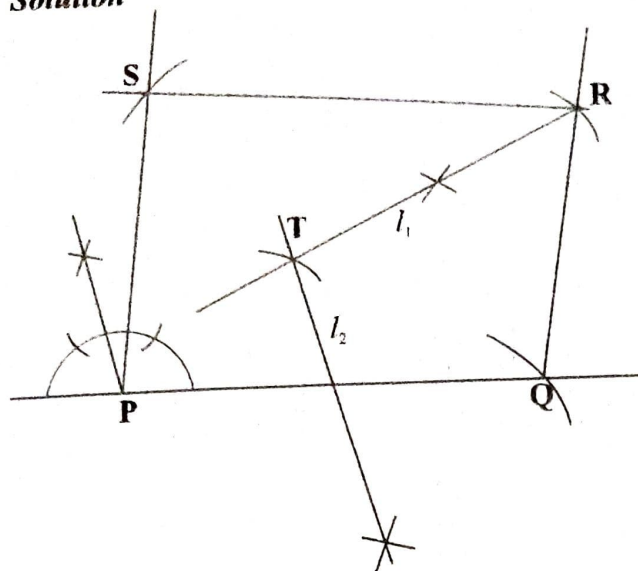
Solution

a) Completed table is shown below.

x	-1	0	1	2	3	4	5	6
y	11	5	1	-1	-1	1	5	11

c) From the graph,
   i) minimum value of $y = -1.3$
   ii) roots are x 1.4, 3.6

iii) $x^2 + 2x + 5 = 7x + 2$

$\Rightarrow x^2 - 5x + 3 = 0$

$\Rightarrow y - 2 = 0$

$\Rightarrow y = 2$

From the graph,

$y = 2 \Rightarrow x = 0.7,\ 4.3$

Question 10

The table shows the distribution of 40 students in a class according to their clubs and the corresponding sectoral angles.

Club	No of students	Sectoral angle
Debating	10	90°
Cultural	x	$(7y)^\circ$
Literacy	14	$(18x)^\circ$
Red Cross	y	81°

a) Find the value of x and y.

b) Illustrate the data on a pie chart.

c) Find the percentage of students who were in the cultural club.

Solution

a) Number of students = 40

$\Rightarrow 10 + x + 14 + y = 40$

$\Rightarrow y = 16 - x \ldots\ldots\ldots\ldots\ldots\ldots(1)$

Also $90 + 7y + 18x + 81 = 360$

$\Rightarrow 7y + 18x = 189 \ldots\ldots\ldots\ldots\ldots(2)$

Putting (1) into 2

$\Rightarrow 7(16 - x) + 18x = 189$

$\Rightarrow 112 - 7x + 18x = 189$

$\Rightarrow x = \dfrac{77}{11} = 7$

From (1) $y = 16 - 7 = 9$

$\therefore x = 7$ and $y = 9$

b) Cultural club $= 7 \times 9 = 63^\circ$

Literacy club $= 18 \times 7 = 126^\circ$
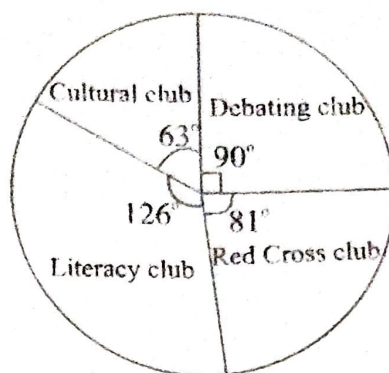
c) Percentage of students who were on the cultural club

$= \dfrac{7}{40} \times 100 = 17.5\%$ **OR**

$= \dfrac{63}{360} \times 100 = 17.5\%$

Question 11

a) It was observed that the shadow of a vertical pole was 6 m longer when the angle of elevation of the sun was 30° than when it was 60°. By means of a sketched diagram, calculate, correct to **two** decimal places, the height of the pole.

b) The length of each non-parallel sides of a trapezium is 18 m while the parallel sides are 32 and 20 m long, respectively. Calculate, correct to the **nearest** degree, the angle which one of the non-parallel sides makes with the shorter of the parallel sides.

Solution

a)
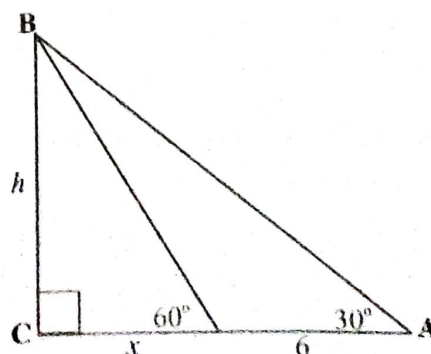
Let the height of the pole be h

From the diagram,

$$\tan 60° = \frac{h}{x}$$

$$\Rightarrow h = x \tan 60° \dots\dots\dots\dots\dots(1)$$

Also $\tan 30° = \dfrac{h}{x+6}$

$$\Rightarrow h = (x+6) \tan 30° \dots\dots\dots(2)$$

Equating (1) and (2)

$$\Rightarrow x \tan 60° = (x+6) \tan 30°$$

$$\Rightarrow 1.7321x = 0.5774x + 3.4641$$

$$\Rightarrow x = \frac{3.4641}{1.1547} = 3\,m$$

From (1) $h = 3 \tan 60° = 5.2\,m$

b)

From the diagram,

$$\theta = 90° + \alpha$$

$$\sin \alpha = \frac{6}{18} \qquad \Rightarrow \alpha = \sin^{-1}\left(\frac{1}{3}\right) = 19.47°$$

$$\Rightarrow \theta = 90° + 19.47° = 109°$$

Question 12

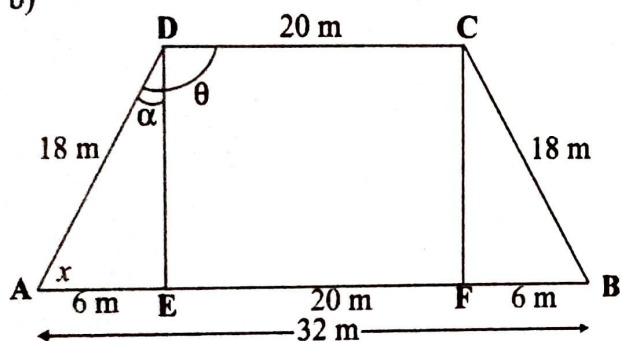The bearing of points X and Y from Z are $040°$ and $300°$, respectively. If $/XY/ = 19.5$km and $/YZ/ = 11.5$ km.

a) Illustrate the information in a diagram,

b) Calculate, correct to the **nearest** whole number,

i) $\angle ZXY$;

ii) $|XZ|$;

Solution

a)
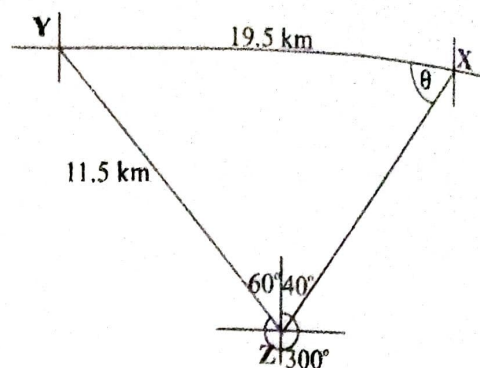
b) i) Using the sine rule

$$\Rightarrow \frac{\sin 100}{19.5} = \frac{\sin \theta}{11.5}$$

$$\Rightarrow \sin \theta = \frac{\sin 100}{19.5} \times 11.4 = 0.5808$$

$$\Rightarrow \theta = \sin^{-1}(0.5808) = 36°$$

ii) $\angle XYZ = 180 - (100 + 35.51) = 44.49°$

Again using the sine rule

$$\Rightarrow \frac{|XZ|}{\sin 44.49} = \frac{19.5}{\sin 100}$$

$$\Rightarrow |XZ| = \frac{19.5}{\sin 100} \times \sin 44.49 = 14\,km$$

Question 13

a) A binary operation $\otimes$ is defined on the set of real numbers, R, by

$$m \otimes n = mn - n - 2m, \text{ where}$$

$m, n \in R$. If $5 \otimes x = 22$, find the value of x.

b) Given that $P(2, -3)$ is a vertex of a triangle

$$PQR, \quad \overrightarrow{PQ} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{RP} = \begin{pmatrix} -4 \\ -1 \end{pmatrix},$$

i) find

α) the coordinates of Q and R;

β) $|\overrightarrow{QR}|$.

ii) If M is the midpoint of $\overrightarrow{PR}$, find $\overrightarrow{MQ}$.

Solution

a) $m \otimes n = mn - n - 2$

$5 \otimes x = 5x - x - 2(5) = 4x - 10$

But $5 \otimes x = 22$

$\Rightarrow 4x - 10 = 22$

$\Rightarrow x = \dfrac{32}{4} = 8$

b) i) α) $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{RP} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$\Rightarrow \overrightarrow{OQ} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\Rightarrow$ Point Q is $Q(5, -1)$

$\overrightarrow{RP} = \overrightarrow{OP} - \overrightarrow{OR}$

$\Rightarrow \overrightarrow{OR} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

$\Rightarrow$ Point R is $R(6, -2)$

β) $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$

$= \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\Rightarrow \left| \overrightarrow{QR} \right| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

ii) $P(2, -3)$ and $R(6, -2)$

$M = \left(\dfrac{2+6}{2}, \dfrac{-3-2}{2} \right) = \left(4, -\tfrac{5}{2}\right)$

$\overrightarrow{MQ} = \overrightarrow{OQ} - \overrightarrow{OM}$

$= \begin{pmatrix} 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -\tfrac{3}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1\tfrac{1}{2} \end{pmatrix}$

38 NOV. 2016 OBJECTIVE TEST

1. In a group of people who collected stamps (S), coins (C) or beads (B) as their hobbies. 15 collected stamps, 5 collected coins and 6 collected beads. Everyone who collected coins also collected stamps. Five people collected both stamps and beads and two people collected all the three items. Which of the following Venn diagrams illustrates the given information?

A.   B.

C.   D.
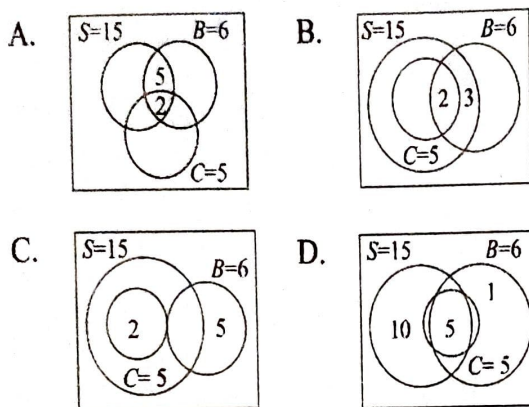
2. Given that x is inversely proportional to y and $x = 9$ when $y = 6$, find x when $y = 18$.
   A. 2   B. 3
   C. 4   D. 5

3. A car travels 245 km at a constant speed in $3\frac{1}{2}$ hours. How far does it travel in 90 minutes?
   A. 47 km   B. 70 km
   C. 105 km   D. 140 km

4. Find the product of 1101_{two} and 111_{two}.
   A. 1101011_{two} B. 1011101_{two}
   C. 1110011_{two} D. 1011011_{two}

5. Correct 0.006586 to **three** significant figures.
   A. 0.00658   B. 0.00659
   C. 0.0066   D. 0.007

6. If p = Musa is short,
   q = Musa is brilliant,
   write, in **symbolic** form, the statement "Musa is short but **not** brilliant."
   A. $p \vee q$ B. $p \vee \sim q$
   C. $p \wedge \sim q$ D. $p \wedge q$

7. The simple interest on ₦80,000.00 for 5 years 3 months is ₦10,500.00. At what rate per annum was the interest charged?
   A. 2.4%   B. 2.5%
   C. 3.3%   D. 3.5%

8. If $x - y = 1$ and $5x - 2y = -1$, evaluate $(x+y)$.
   A. –3   B. –1
   C. 1   D. 3

9. If $T = \dfrac{a-m}{1+am}$, find a in terms of T and m.
   A. $a = \dfrac{Tm}{1-Tm}$ B. $a = \dfrac{T+m}{1+Tm}$
   C. $a = \dfrac{T-m}{1+Tm}$ D. $a = \dfrac{T+m}{1-Tm}$

10. Solve: $\frac{2}{3}(x-1) > \frac{1}{2}(1-2x) + \frac{1}{2}$.
    A. $x > 10$ B. $x > 1$
    C. $x > -1$ D. $x > -10$

11. If $\dfrac{2}{3} + \dfrac{2}{b} = \dfrac{1}{c}$ and $c = 3\dfrac{3}{4}$, find the value of b.
    A. 15   B. $2\frac{1}{7}$
    C. $\dfrac{4}{5}$ D. –5

12. A box containing 12 plates and 6 cups costs GH¢156.00. Another box containing 12 plates and 18 cups of the same type costs GH¢228.00. Find the cost of 9 plates.
    A. GH¢70.00   B. GH¢80.00
    C. GH¢90.00   D. GH¢100.00

13. Factorise: $7 - 6x - x^2$.
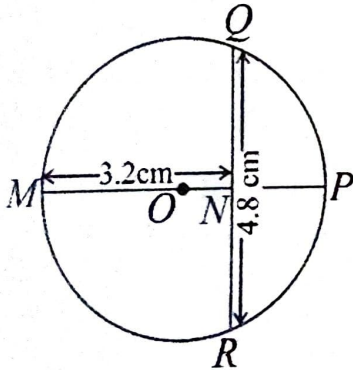
A. $(x - 7)(1 - x)$ B. $(x - 1)(x - 7)$

C. $(x + 7)(1 - x)$ D. $(x - 1)(x + 7)$

14. The volume of a cone with base radius 4 cm is 201 cm³. Calculate, correct to the nearest whole number, the height of the cone.

[Take $\pi = \frac{22}{7}$].

A. 10 cm   B. 11 cm

C. 12 cm   D. 13 cm

15. In the diagram, $\overline{MP}$ is a diameter of the circle centre O. $\overline{QR}$ is a chord which is perpendicular to the diameter at N. If $|QR| = 4.8$ cm and $|MN| = 3.2$ cm, calculate the radius of the circle.

A. 0.7 cm   B. 2.0 cm

C. 2.5 cm   D. 3.0 cm

16. The total surface area of a cube is 24 cm². What is the volume of the cube?

A. 8 cm³   B. 24 cm³

C. 64 cm³   D. 486 cm³

17. The perimeter of a rectangular floor is 26 m. Its length is x m. Find the area of the floor in m².

A. $x(x + 13)$ B. $x(x - 13)$

C. $x(13 - x)$ D. $x(26 - x)$

18. A straight line which divides one or more lines and angles into two equal parts is called a

A. transversal.   B. divisor.

C. tangent.   D. bisector.

19.
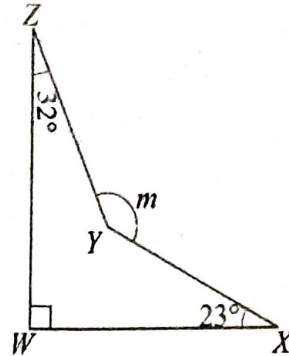
In the diagram, $\angle WZY = 32°$, $\angle YXW = 23°$, $\angle ZWX$ is a right angle and $\angle ZYX = m$. Find the value of m.

A. 135°   B. 145°

C. 154°   D. 165°

20. In the diagram, $PR // VS$, $|QU| = |QT|$ and $\angle UQT = 70°$.

Find $\angle PQU$.

A. 55°   B. 65°

C. 70°   D. 84°

21.
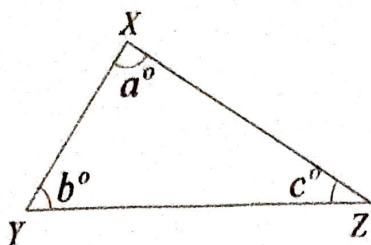
In the diagram, $a : b : c = 4 : 3 : 2$.
Find $(a - c)^\circ$.

A. 20°

B. 40°

C. 60°

D. 80°

22.
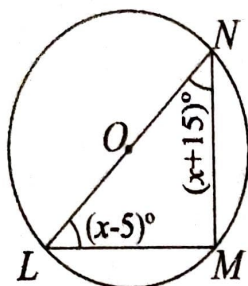
The diagram shows a triangle inscribed in a circle of centre O. If $\angle LNM = (x+15)^\circ$ and $\angle NLM = (x-5)^\circ$, find the value of x.
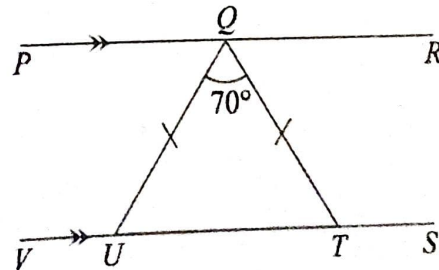
A. 35°

B. 40°

C. 45°

D. 50°

23.
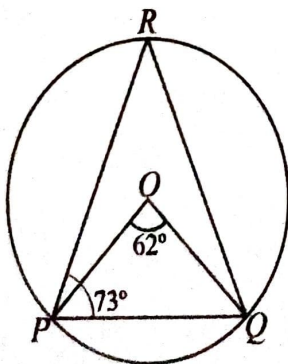
In the diagram, O is the centre of the circle PQR. $\angle RPQ = 73^\circ$ and $\angle POQ = 62^\circ$.
Calculate $\angle RQO$.

A. 11°

B. 14°

C. 17°

D. 31°

24. What is the equation of the line which passes through the points $P(3, 5)$ and $Q(-1, 3)$?

A. $2y - x - 7 = 0$

B. $2y + x - 1 = 0$

C. $y - 2x - 5 = 0$

D. $2y + x - 7 = 0$

25. The equation of a line is $-3x + y = 1$. What is the gradient of the line?

A. 3

B. 1

C. -1

D. -3

26.
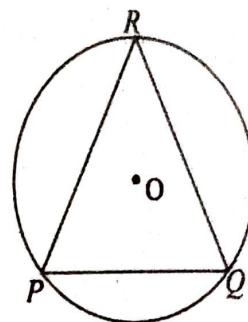
In the diagram, PQR is a triangle inscribed in a circle of centre O. If $|PQ| = 12$ cm and O, the centre of the circle, is 5 cm from PQ, calculate, correct to the nearest degree, $\angle PRQ$.

A. 25°

B. 50°

C. 75°

D. 100°

27. Sammy moved from point **M**, 6 km due west to point **K** and then 6 km on a bearing of 300° to point **L**. What is the bearing of **L** from **M**?

A. 300°

B. 285°

C. 270°

D. 195°

28. If $\cos x = \dfrac{1}{\sqrt{2}}$, $0^\circ \le x \le 90^\circ$, evaluate

$2\tan x + \sqrt{2}\sin x$.

A. 3

B. $2 + \frac{\sqrt{2}}{2}$

C. 2

D. $1 + \frac{\sqrt{2}}{2}$

29. Find the probability of picking the letter **T** from the word **STUDENT**.

A. $\frac{1}{7}$

B. $\frac{1}{6}$

C. $\frac{2}{7}$

D. $\frac{1}{3}$

30. The table gives the distribution of marks obtained by a number of pupils in a class test.

Marks	0	1	2	3	4	5
Frequency	4	7	12	18	11	8

Find the mode of the distribution.

A. 3   B. 5   C. 8   D. 18

31. In a class of 45 students, 28 offer Chemistry and 25 Biology. If each of the students offers **at least** one subject, what is the probability that a student chosen at random from the class offers Chemistry only?

A. $\frac{28}{45}$

B. $\frac{5}{9}$

C. $\frac{4}{9}$

D. $\frac{17}{45}$

32. Find the range of these fractions:

$\frac{3}{4}, \frac{1}{2}, \frac{3}{5}$ and $\frac{2}{3}$.

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

33. The following are arranged in order of size: $y - 2, y + 2, 4, 2y + 18$. If the median equals the mean, find the value of y.

A. -5

B. -4

C. -3

D. -2

34. Express $a \log_y b = x$ in index form.

A. $a^y = b^x$

B. $y^x = a^b$

C. $y^x = b^a$

D. $y^b = a^x$

35. Solve: $2^{3x} = 0.25$

A. $x = -1\frac{1}{2}$

B. $x = -\frac{2}{3}$

C. $x = \frac{2}{3}$

D. $x = 1\frac{1}{2}$

36. Simplify: $\left(\frac{1}{16}\right)^{-\frac{1}{2}} + \left(\frac{8}{27}\right)^{\frac{2}{3}}$.

A. $\frac{13}{36}$

B. $\frac{25}{36}$

C. $1\frac{7}{9}$

D. $4\frac{4}{9}$

37. The sum of the interior angles of a polygon is three times the sum of its exterior angles. How many sides has the polygon?

A. 4   B. 6   C. 8   D. 12

38. Find the values of x for which $\dfrac{x-4}{x^2-2x-3}$ is undefined.

A. -1 and -3

B. 1 and 3

C. 1 and -3

D. -1 and 3

39. If $\dfrac{\sqrt{2}}{2}(1-\sqrt{2})^2$ is expressed in the form $a+b\sqrt{2}$, find $(a+b)$

A. $-3\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $3\frac{1}{2}$

40. If $\dfrac{2}{x} = \dfrac{2x-1}{3}$, find x.

A. $-2, 1\frac{1}{2}$

B. $-2, \frac{2}{3}$

C. $2, -\frac{2}{3}$

D. $2, -1\frac{1}{2}$

41. Evaluate $6 - 36(\text{mod } 9)$.

    A. 3   B. 4

    C. 5   D. 6

42. The nth term of a sequence is $n^2 - n - 2$. Find the sum of the first and third terms.

    A. –2   B. 2

    C. 4   D. 6

43. The total surface area of a solid cylinder is 484 cm². If its base has a diameter of 14 cm, find its height.   [Take $\pi = \frac{22}{7}$]

    A. 4 cm   B. 6 cm

    C. 7 cm   D. 8 cm

44. The ratio of the length to the width of a rectangle is 5 : 4. If its perimeter is 54 cm, find the length.

    A. 30 cm   B. 24 cm

    C. 18 cm   D. 15 cm

45. Use the cumulative frequency curve to estimate the 80th percentile.
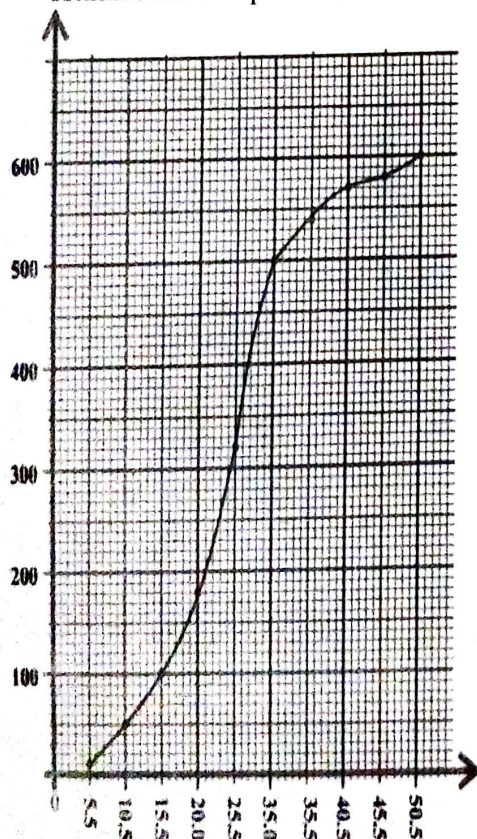
    A. 30.5   B. 30.0

    C. 29.5   D. 29.0

46. Simplify: $\dfrac{\frac{1}{4}m^2 - 9n^2}{\frac{1}{4}\left(\frac{1}{2}m - 3n\right)}$.

    A. $\frac{1}{4}\left(\frac{1}{2}m + 3n\right)$ B. $4\left(\frac{1}{2}m - 3n\right)$

    C. $\frac{1}{4}\left(\frac{1}{2}m - 3n\right)$ D. $4\left(\frac{1}{2}m + 3n\right)$

47. The bar chart shows the marks distribution of students in an English test. What percentage of the students scored between 35 and 50 marks?
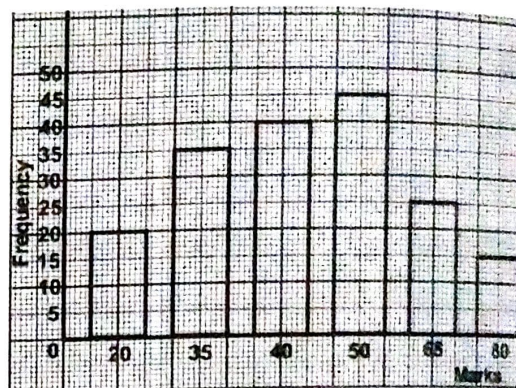
    A. $55\frac{1}{3}\%$ B. 60%

    C. 65 %   D. $66\frac{2}{3}\%$

48. In the figure, O is the centre of the circle SRP, $\angle ROS = 110°$ and $\angle QRP$ is a right angle. Find the value of x.

    A. 70   B. 55

    C. 40   D. 35
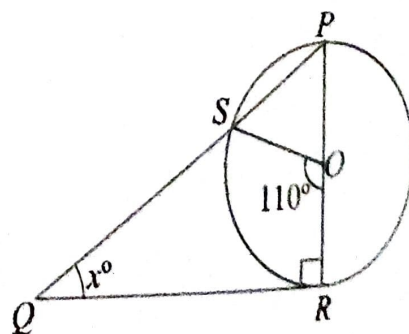
49. In the figure, *PQRS* is a cyclic quadrilateral, $|QR| = |SR|$, $\angle SPT = x°$ and $\angle QSR = 37°$. Find the value of *x*.

A. 143   B. 106
C. 90   D. 74
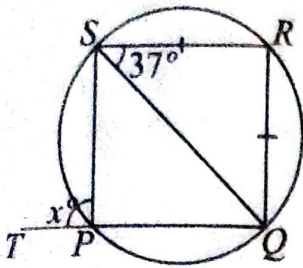
50. The diagram is a graph of the relation $y = x^2 - 5x + 4$. Find the range of values of *x* for which *y* decreases as *x* increases.

A. $-1 \le x \le 1$ B. $-1 \le x \le 2.5$
C. $1 \le x \le 4$ D. $2.5 \le x \le 6$

NOV. 2016 THEORY QUESTIONS AND ANSWERS

Question 1

a) Solve: $\dfrac{3x+1}{4} - \dfrac{3+4x}{3} \le 1$.

b) i) Given that: $\dfrac{1}{x} + \dfrac{2}{y} = \dfrac{1}{z}$, express y in

   terms of x and z.

   ii) If $x = -5$ and $z = 10$, find the value of y, leaving the answer as a mixed number.

Solution

a) $\dfrac{3x+1}{4} - \dfrac{3+4x}{3} \le 1$.

Multiplying through by 12 gives:

$3(3x + 1) - 4(3 + 4x) \le 12$

$9x + 3 - 12 - 16x \le 12$

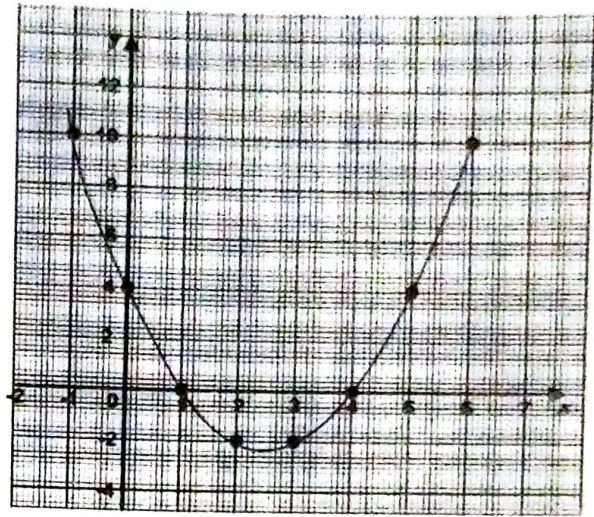$9x - 16x \le 12 - 3 + 12$

$-7x \le 21$

$\dfrac{-7x}{-7} \ge \dfrac{21}{-7}$

$x \ge -3$

b) i) $\dfrac{2}{y} = \dfrac{1}{z} - \dfrac{1}{x} \quad \Rightarrow \quad \dfrac{2}{y} = \dfrac{x-z}{xz}$

$\Rightarrow 2xz = y(x-z)$

$\therefore y = \dfrac{2xz}{x-z}$

ii) $y = 6\frac{2}{3}$

Question 2

A man bought 250 oranges for D 1,000.00. He kept 20% of the oranges for himself, sold 115 at D 6.50 each and the rest at D 5.00 each. Calculate his percentage profit.

Solution

He kept 20% of the oranges

So $\dfrac{20}{100} \times 250 = 50$ oranges were kept

Thus $250 - 50 = 200$ oranges are left

He sold 115 at D 6.50 each

This implies $(115 \times D 6.50) = $ D 747.50 sales

Remainder $= (200 - 115)$ oranges $= 85$ oranges

He sold the rest at D 5.00 each

$\Rightarrow 85 \times D 5.00 = D 425.00$ sales

Total sales $= $ D 747.50 + D 425.00 = D 1,172.50

Profit $= $ (Sales) $-$ (Cost) = D 1,172.50 $-$ D1,000

Hence Profit $= $ D 172.50

Percentage profit $= \dfrac{\text{Profit}}{\text{Cost}} \times 100\%$

So percentage profit $= \dfrac{172.50}{1000} \times 100\%$

$\therefore$ Percentage profit $= 17.25\%$

Question 3
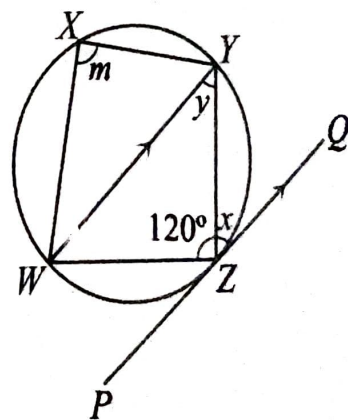
In the diagram, PQ is a tangent to the circle at Z. If $PQ//WY$, $\angle WZY = 120°$, $\angle WXY = m$, $\angle WYZ = y$ and $\angle YZQ = x$, find the value of:

a) m; \qquad b) x.

Solution

a) Opposite angles of a cyclic quadrilateral add up to 180° (i.e. they are supplementary)

So $\angle WZY + \angle WXY = 180°$

$m + 120° = 180°$ \qquad $\therefore m = 60°$

b) $PQ \parallel WY$

$\therefore y = x$ (alternate angles)

Since angles in alternate segments are equal, $\angle ZWY = x$

$x + x + 120° = 180°$

$2x + 120° = 180° \implies 2x = 60°$

$\therefore x = 30°$

Question 4

A box contains five blue, three red and two white identical balls. If 2 balls are selected at random, one after the other with replacement, from the box, find the probability of selecting:

a) two blue or two red balls;

b) one red and one white ball.

Solution

a) Total number of balls $= 5 + 3 + 2 = 10$

Probability of selecting a blue ball $= P(B) = \frac{5}{10}$

Probability of selecting a white ball $= P(W) = \frac{2}{10}$

Probability of selecting a red ball $= P(B) = \frac{3}{10}$

P(two blue or two red balls)

$[P(B) \times P(B)] + [P(R) \times P(R)]$

$= \left(\frac{3}{10} \times \frac{3}{10}\right) + \left(\frac{5}{10} \times \frac{5}{10}\right) = \frac{17}{50} = 0.34$

b) P(one red and one white ball)

$= \left(\frac{3}{10} \times \frac{2}{10}\right) + \left(\frac{3}{10} \times \frac{2}{10}\right) = \frac{3}{50} + \frac{3}{50}$

$\therefore$ P(one red and one white ball) $= \frac{3}{25} = 0.12$

Question 5
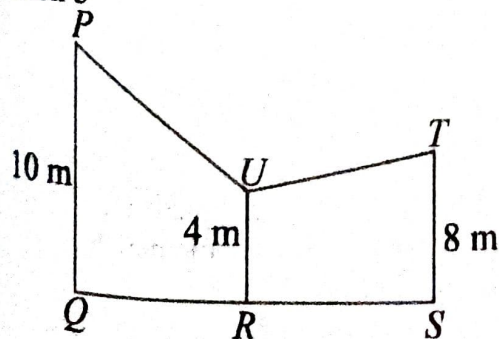
In the diagram, PQ, UR and TS are poles which are on the same horizontal ground. $|PQ| = 10$ m, $|UR| = 4$ m and $|TS| = 8$ m.

If the angles of elevation of P and T from U are 51° and 20° respectively, find, correct to 2 decimal places, the:

a) distance between Q and S;

b) angle of depression of S from U.

Solution

a) The given diagram can be modified as follows:
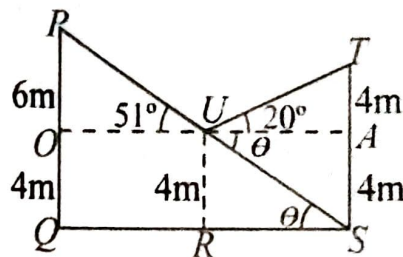
From the diagram, $\tan 51° = \dfrac{|OP|}{|OU|}$

$\therefore |OU| = \dfrac{|OP|}{\tan 51°} = \dfrac{6}{\tan 51°} = 4.8587$ m

Also from the diagram, $\tan 20° = \dfrac{|AT|}{|AU|}$

$\therefore |AU| = \dfrac{|AT|}{\tan 20°} = \dfrac{4}{\tan 20°} = 10.9899$

Now $|RS| = |AU| = 10.989$ m

Also $|QR| = |OU| = 10.989$ m

$|QS| = |QR| + |RS| \implies |QS| = 4.859 + 10.989$

$\therefore |QS| = 15.85$ m (2 d.p)

b) Let θ be the angle of depression of S from U, as shown in the following figure:
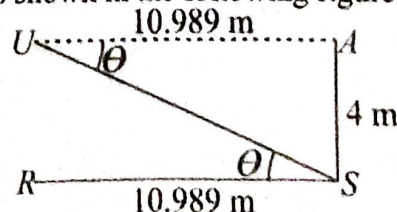
From the diagram, $\tan\theta = \dfrac{AS}{RS}$

$\tan\theta = \dfrac{4}{10.989}$ $\qquad \theta = \tan^{-1}\left(\dfrac{4}{10.989}\right)$

$\therefore \theta = \tan^{-1}(0.364) = 20.00°$

Hence angle of depression of S from U is 20.00°.

Question 6

a) If $2^x + 2^{(x-1)} = 48$, find the value of x.

b) A worker is given a tax-free allowance of GH¢5,000.00 and he pays 20 pesewas in the Ghana cedi as tax on the rest of his income. If his net income is GH¢21,000.00, calculate the:
   i) taxable income;
   ii) income tax.

Solution

a) Method 1

$2^x + 2^{(x-1)} = 48 \qquad \Rightarrow 2^x + 2^x \times 2^{-1} = 48$

$2^x + 2^x \times \frac{1}{2} = 48 \qquad \Rightarrow 2^x + \dfrac{2^x}{2} = 48$

$2^x(1+\frac{1}{2}) = 48 \qquad \Rightarrow 2^x(\frac{3}{2}) = 96$

$2^x(3) = 96 \qquad\qquad \Rightarrow 2^x = \dfrac{96}{3}$

$2^x = 32 \qquad\qquad\quad \Rightarrow 2^x = 2^5$

$\therefore x = 5$

Method 2

$2^x + 2^{(x-1)} = 48 \qquad \Rightarrow 2^x + \dfrac{2^x}{2^1} = 48$

Multiplying through by 2 gives:

$2(2^x) + 2\left(\dfrac{2^x}{2}\right) = 2(48)$

$(2)(2^x) + 2^x = 96 \qquad \Rightarrow 2^x(2+1) = 96$

$2^x(3) = 96 \qquad\qquad \Rightarrow 2^x = \dfrac{96}{3}$

$2^x = 32 \qquad \Rightarrow 2^x = 2^5 \qquad \therefore x = 5$

b) Let the gross income (or gross salary) $= x$

Now Income tax $= \dfrac{20}{100}(x - 5,000)$

But Net salary = (Gross salary) – (Income tax)

$21,000 = x - \dfrac{20}{100}(x - 5,000)$

$21,000 = x - 0.2(x - 5000)$

$21,000 = x - 0.2x + 1,000$

$21,000 - 1,000 = 0.8x$

$\therefore x = \dfrac{20,000}{0.8} = 25,000$

i) Taxable income
   = (Gross salary) – (Tax-free income)
   = 25,000 – 5,000 = GH¢ 20,000.00

ii) Income tax $= \dfrac{20}{100}(x - 5,000) = \dfrac{20}{100} \times 20,000$

$\therefore$ Income tax = GH¢4,000.00

Question 7

a) Solve: $3\log_{10} 2 - 2\log_{10}3 = 1 + \log_{10}\left(\frac{1}{x}\right)$

b) An engineer walked round a cylindrical petrol container 8 m high once, keeping a constant distance of 1 m from the container. If he walked with a speed of 3 km/hr for 3 minutes, calculate, correct to the nearest whole number, the:
   i) radius of the container;
   ii) volume of the container.
   [Take $\pi = \frac{22}{7}$]

Solution

a) $3\log_{10} 2 - 2\log_{10}3 = 1 + \log_{10}\left(\frac{1}{x}\right)$

$\log_{10} 2^3 - \log_{10} 3^2 = \log_{10}10 + \log_{10} 1 - \log_{10} x$

$\log_{10} 8 - \log_{10} 9 = \log_{10}10 + 0 - \log_{10} x$

$\log_{10}\left(\dfrac{8}{9}\right) = \log_{10}\left(\dfrac{10}{x}\right)$
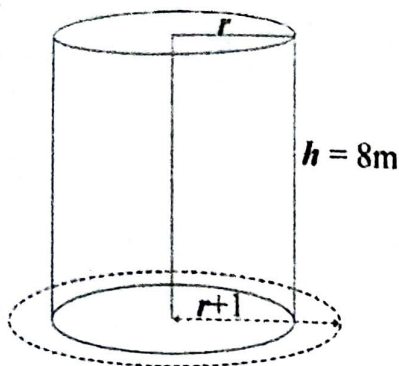
$\dfrac{8}{9} = \dfrac{10}{x}$ $8x = 90$

$x = \dfrac{90}{8}$ $x = \dfrac{45}{4}$

$\therefore x = 11\tfrac{1}{4} = 11.25$

b) i) The information can be illustrated on a diagram as follows:

Speed = 3 km/hr = $\dfrac{3 \times 1000}{60 \times 60}$ m/s = $\dfrac{5}{6}$ m/s

Time = 3 minutes = 3×60 s = 180 s

Distance covered by the engineer

= Speed × time = $\dfrac{5}{6} \times 180 = 150$ m

Let r = radius of the cylindrical container;
From the diagram, radius of circle = $(r + 1)$ m,
∴ Circumference of the circle

= $2\pi(r + 1) = 2 \times \dfrac{22}{7}(r + 1) = \dfrac{44}{7}(r + 1)$

But circumference of the circle = Distance covered by the engineer

So $\dfrac{44}{7}(r + 1) = 150$

$44(r + 1) = 7 \times 150$ $44r + 44 = 1050$

$44r = 1006$ $\therefore r = \dfrac{1006}{4} = 22.86$

Hence $r = 23$ m (to the nearest whole number)

ii) Volume of container

= $\pi r^2 h = \dfrac{22}{7} \times 22.86^2 \times 8 = 13{,}139.144$ cm^3

∴ Volume of cylinder = 13,139 m^3

Question 8

An aeroplane flies at an average speed of 950 km/hr from town P (Lat. 40° S, Long. 29.5° W) due east to town Q and then due south to town R. If the distance from Q to R along their common longitude is 4,500 km and the whole journey took 11 hours, calculate, correct to one decimal place, the:
a) longitude of Q;
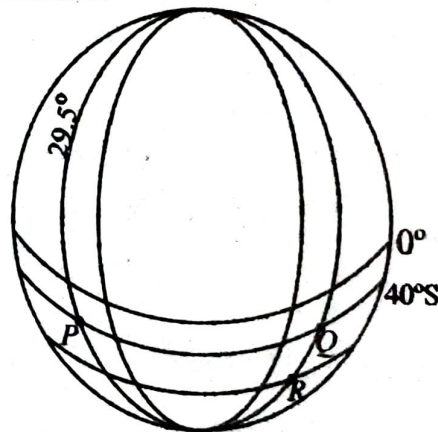b) latitude of R.
[Take radius of the earth = 6,400 km, $\pi = 3.142$]

Solution

a) The following diagram is an illustration of the situation:

$|PQ| = 950 \times (11 - 4.73684)$
$|PQ| = 5950$ km

$5950 = \dfrac{x}{360} \times 2 \times 3{,}142 \times 6400 \times \cos 40°$

$69.526 - 29.5°$
Longitude of $Q = x = 40.0$ °E

b) $4500 = \dfrac{y}{360} \times 2 \times 3.142 \times 6400$

$$v = \frac{4500 \times 360}{2 \times 1.142 \times 6400} = 40.28°$$

But $40.28° + 40° = 80.28°$

Latitude of $R = v = 80.3°S$

Question 9

a) Using ruler and a pair of compasses only, construct a

i) square $ABCD$ of side 7 cm and shade it;

ii) circle to pass through the vertices of the square

b) Using the construction, calculate the perimeter of the circle.

Solution

a) (i) and (ii)

Construct line $|AB| = 7$ cm. Construct two perpendicular lines to meet line AB; through A and B. Using a radius of 7 cm and with A and B as centres, draw two arcs to cut the two perpendicular lines respectively. Label the points of intersection of the arcs and the perpendicular bisectors as C and D.

With B as the centre, bisect $\angle ABC$ and with C as the centre, bisect $\angle BCD$. Locate the point of intersection of the two angle bisectors and label it as O. With O as the centre, draw a circle through A, B, C and D.

The construction is as follows:
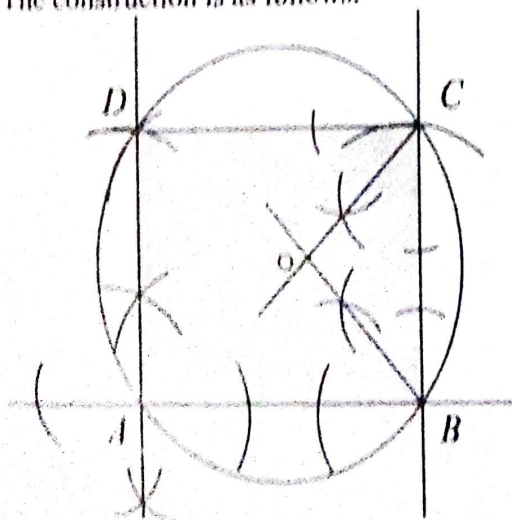
b) $|OC| = 5$ cm $\Rightarrow$ Radius of circle is $r = 5$ cm

Perimeter of circle

$$= 2\pi r = 2 \times \frac{22}{7} \times 5 = \frac{220}{7} = 31.43 \text{ cm}$$

Question 10

a) Copy and complete the following table of values for the relation $y = x^2 - 2x - 5$ for $-3 \leq x \leq 5$.

x	-3	-2	-1	0	1	2	3	4	5
y			-2					-2	10

b) Using scales of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw the graph of the relation $y = x^2 - 2x - 5$ for $-3 \leq x \leq 5$.

c) On the same axes, draw the graph of $y = 2x - 3$

d) Using the graphs, find the coordinates of the points of intersection.

Solution

a) The given curve is: $y = x^2 - 2x - 5$.

When $x = -3$, $y = (-3)^2 - 2(-3) - 5 = 10$
When $x = -2$, $y = (-2)^2 - 2(-2) - 5 = 3$
When $x = 0$, $y = (0)^2 - 2(0) - 5 = -5$
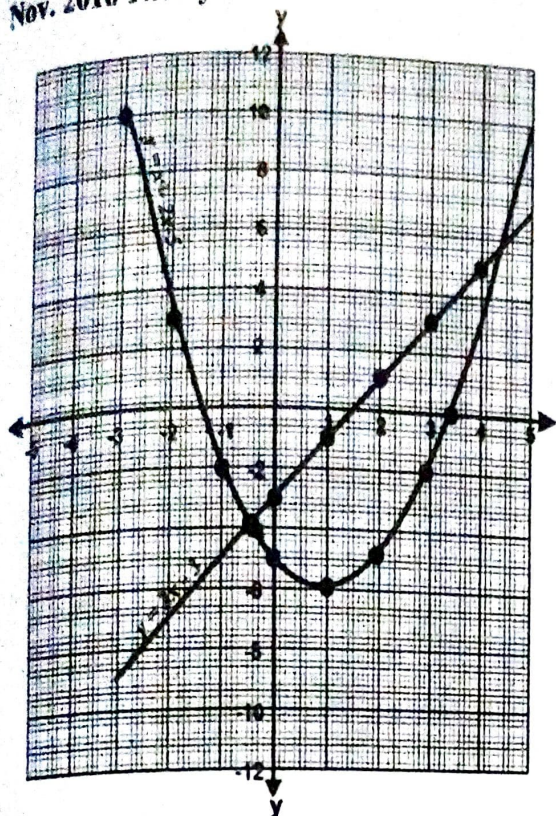When $x = 1$, $y = (1)^2 - 2(1) - 5 = -6$
When $x = 2$, $y = (2)^2 - 2(2) - 5 = -5$
When $x = 4$, $y = (4)^2 - 2(4) - 5 = 3$

The complete table of values is as follows:

x	-3	-2	-1	0	1	2	3	4	5
y	10	3	-1	-5	-6	-5	-2	3	10

b) The graph is as follows:

c) For $y = 2x - 3$, when $x = 0$, $y = -3$
So the point $(0, -3)$ lies on the line.

When $y = 0$, $0 = 2x - 3$ giving us $x = \dfrac{3}{2} = 1.5$

So the point $(1.5, 0)$ lies on the line.
The line has been drawn on the graph in (b).

d) From the graph, the points of intersection
are: $(-0.4, -3.8)$ or $(4.4, 5.6)$

Question 11

a) Given that $\sin x = \frac{1}{2}$, where $0° \leq x \leq 90°$,

evaluate $\dfrac{\sin x \cos x}{\cos x + \tan x}$

b)
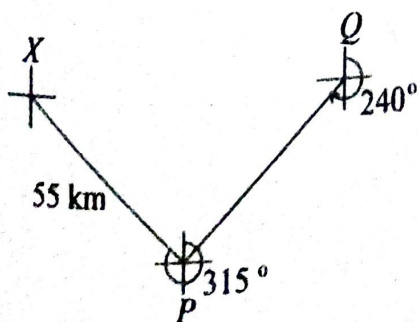
The diagram shows the position of three points
P, Q and X on a horizontal plane. The bearing
of P from Q is $240°$ and that of X from P is
$315°$. If $|PQ| = 36$ km, and $|PX| = 55$ km,
calculate, correct to one decimal place,

i) $|QX|$;

ii) the bearing of Q from X.

Solution

a) $\sin x = \frac{1}{2}$ is illustrated on the following
diagram:
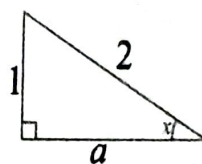
From Pythagoras theorem,
$a^2 + 1^2 = 2^2 \implies a^2 = 4 - 1 = 3 \implies a = \sqrt{3}$

From the figure, $\cos x = \dfrac{a}{2} = \dfrac{\sqrt{3}}{2}$

Also, $\tan x = \dfrac{1}{a} = \dfrac{1}{\sqrt{3}}$

$\therefore \dfrac{\sin x \cos x}{\cos x + \tan x} = \dfrac{\frac{1}{2} \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}} = \dfrac{\frac{1}{4}\sqrt{3}}{\frac{5}{2\sqrt{3}}} = \dfrac{3}{10} = 0.3$

b) Let $\angle PXQ = \theta$ as shown in the following
diagram.
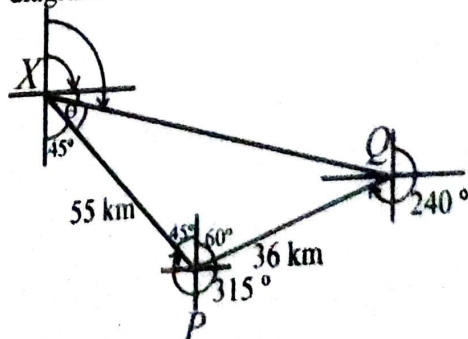
i) Applying the cosine rule, we have
$|QX|^2 = 36^2 + 55^2 - 2(55)(36)\cos 105°$

$$|QX|^2 = 4321 + 1024.923 = 5345.923$$

$$|QX| = \sqrt{5345.923} = 73.1 km$$

ii) Applying the sine rule, we have

$$\frac{36}{\sin\theta} = \frac{73.1158}{\sin 105°} \Rightarrow \sin\theta = \frac{36\sin 105}{73.1158} = 0.4756$$

$$\therefore \theta = \sin^{-1}(0.4756) = 28.398°$$

Hence the bearing of Q from X

$$= 180 - (45 + \theta) = 180 - (45 + 28.398) = 106.6°$$

Question 12

a) Using a scale of 2 cm to 2 units on each axis, draw, on a sheet of graph paper, two perpendicular axes $0x$ and $0y$ for $-10 \le x \le 10$ and $-12 \le y \le 12$.

b) Draw on this graph, indicating the coordinates of all vertices:

i) quadrilateral $ABCD$ with vertices $A(-5, -4)$, $B(2, -1)$, $C(0, 3)$ and $D(-8, 4)$;

ii) the image $A_1B_1C_1D_1$ of $ABCD$ under a translation by the vector $\binom{3}{-8}$ where $A \to A_1, B \to B_1, C \to C_1, D \to D_1$;

iii) the image $A_2B_2C_2D_2$ of $ABCD$ under an enlargement from the origin with a scale factor $-\frac{1}{2}$, where $A \to A_2, B \to B_2, C \to C_2, D \to D_2$.

c) Find the equation of $\overline{A_1D}$.

Solution

a) Refer to the graph.

b) (i) and (ii)

We have been given the following:

$A(-5, -4)$, $B(2, -1)$, $C(0, 3)$ and $D(-8, 4)$;

$$A\binom{-5}{-4} + \binom{3}{-8} \to A_1\binom{-2}{-12}$$

$$B\binom{2}{-1} + \binom{3}{-8} \to B_1\binom{5}{-9}$$

$$C\binom{0}{3} + \binom{3}{-8} = C_1\binom{3}{-5}$$

$$D\binom{-8}{4} + \binom{3}{-8} = D_1\binom{-5}{-4}$$

iii) $\binom{x}{y} \to \binom{-\frac{1}{2}x}{-\frac{1}{2}y}$

$$A\binom{-5}{-4} \to A_2\binom{-\frac{1}{2}\times5}{-\frac{1}{2}\times-4} = A_2\binom{-2.5}{-2}$$

$$B\binom{2}{-1} \to B_2\binom{-\frac{1}{2}\times2}{-\frac{1}{2}\times-1} = B_2\binom{-1}{0.5}$$

$$C\binom{0}{3} \to C_2\binom{-\frac{1}{2}\times0}{-\frac{1}{2}\times3} = C_2\binom{0}{-1.5}$$

$$D\binom{-8}{4} \to D_2\binom{-\frac{1}{2}\times-8}{-\frac{1}{2}\times4} = D_2\binom{4}{-2}$$

The following is the graph:

c) $A_1(-2, -12)$ and $D(-8, 4)$

Gradient of line A_1D is:

$$m = \frac{-12-4}{-2+8} = \frac{-16}{6} = \frac{-8}{3}$$

Equation of line A_1D is:

$$\frac{y+12}{x+2} = \frac{-8}{3} \implies 3y + 36 = -8x - 16$$

$$\implies 3y + 8x + 52 = 0$$

Question 13
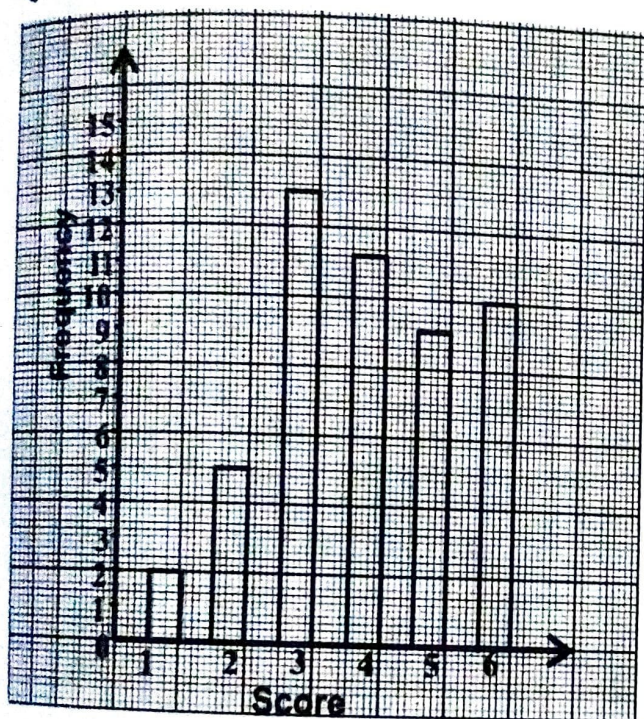
The bar chart represents the outcome when a die is rolled a number of times.

a) Use the bar chart to construct a frequency distribution table;

b) Use the frequency table to calculate the:
   i) mean;
   ii) standard deviation of the distribution.

Solution

a) The frequency distribution table is as follows:

x	f	fx	$(x-\bar{x})$	$f(x-\bar{x})^2$
1	2	2	-2	18
2	5	10	-3	20
3	13	39	-1	13
4	11	44	0	0
5	9	45	1	9
6	10	60	2	40
	$\sum f$ $= 50$	$\sum fx$ $= 200$		$\sum f(x-\bar{x})^2$ $= 100$

b) i) The mean is $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{200}{50} = 4$

ii) **Method 1**

The standard deviation is:

$$s = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{100}{50}} = \sqrt{2} = 1.414$$

Method 2

x	f	fx	fx^2
1	2	2	4
2	5	10	20
3	13	39	117
4	11	44	178
5	9	45	225
6	10	60	360
	$\sum f$ $= 50$	$\sum fx = 200$	$\sum fx^2 = 904$

The standard deviation is

$$s = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{904}{50} - \left(\frac{200}{50}\right)^2}$$

$$\implies s = \sqrt{18.08 - 16} = \sqrt{2.08} = 1.414$$

39 JUNE 2017 OBJECTIVE TEST

1. Express 0.0000407, correct to 2 significant figures.
   A. 0.0
   B. 0.00004
   C. 0.000041
   D. 0.000407

2. If x varies inversely as y and y varies directly as z, what is the relationship between x and y.

   A. $x \propto z$
   B. $x \propto \dfrac{1}{z}$

   C. $x \propto z^2$
   D. $x \propto \dfrac{1}{z^2}$

3. Evaluate $\dfrac{3\frac{1}{4} \times 1\frac{3}{5}}{11\frac{1}{3} - 5\frac{1}{3}}$

   A. $\frac{14}{15}$
   B. $\frac{13}{15}$
   C. $\frac{4}{5}$
   D. $\frac{11}{15}$

Fig. 1

⊕	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Fig. 2

⊗	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

4. Fig. 1 and Fig. 2 are the addition and multiplication tables respectively in modulo 5. Use these tables to solve the equation $(n \otimes 4) + 3 = 0 \pmod 5$.
   A. 1
   B. 2
   C. 3
   D. 4

5. The ages of Tunde and Ola are in the ratio 1: 2. If the ratio of Ola's age to Musa's age is 4: 5, what is the ratio of Tunde's age to Musa's age?
   A. 1:4
   B. 1:5
   C. 2:5
   D. 5:2

6. If $M = \{x : 3 \le x \le 8\}$ and $N = \{x : 8 < x \le 12\}$ which of the following is true?
   I. $8 \in M \cap N$
   II. $8 \in M \cup N$
   III. $M \cap N = \phi$

   A. III only
   B. I and II only
   C. II and III only
   D. I, II and III

7. Given that $a = \log 7$ and $b = \log 2$, express $\log 35$ in terms of a and b.
   A. $a + b + 1$
   B. $ab - 1$
   C. $a - b + 1$
   D. $b - a + 1$

8. If $x = \frac{2}{3}$ and $y = -6$ evaluate $xy - \frac{y}{x}$
   A. 0
   B. 5
   C. 8
   D. 9

9. Solve the equation $\frac{1}{5x} + \frac{1}{x} = 3$.
   A. $\frac{1}{5}$
   B. $\frac{2}{5}$
   C. $\frac{3}{5}$
   D. $\frac{4}{5}$

10. A sum of ₦ 18,100.00 was shared among 5 boys and 4 girls with **each** boy taking ₦ 20.00 more than each girl. Find a boy's share.
   A. ₦ 1,820.00
    B. ₦ 2,000.00
    C. ₦ 2,020.00
    D. ₦ 2,040.00

11. One factor of $7x^2 + 33x - 10$ is
   A. $7x + 5$
   B. $x - 2$
   C. $7x - 2$
   D. $x - 5$

12. Solve $-\frac{1}{4} < \frac{3}{4}(3x - 2) < \frac{1}{2}$.
   A. $\frac{5}{9} < x < \frac{8}{9}$
   B. $-\frac{8}{9} < x < \frac{7}{9}$
   C. $-\frac{8}{9} < x < \frac{5}{9}$
   D. $-\frac{7}{9} < x < \frac{8}{9}$

13. Simplify $3x - (p - x) - (r - p)$
   A. $2x - r$
   B. $2x + r$
   C. $4x - r$
   D. $2x - 2p - r$

14. An arc of a circle of radius 7.5 cm is 7.5 cm long. Find, correct to the **nearest** degree, the angle which the arc subtends at the centre of the circle. [Take $\pi = \frac{22}{7}$]
   A. 29°
   B. 57°
   C. 65°
   D. 115°

15. Water flows out of a pipe at a rate of $40\pi \text{ cm}^3$ per second into an empty cylindrical container of base radius 4 cm. Find the height of water in the container after 4 seconds.
   A. 10 cm
   B. 14 cm
   C. 16 cm
   D. 20 cm

16. The dimensions of a water tank are 13 cm, 10 cm and 70 cm. If it is half-filled with water, calculate the volume of water in litres.
   A. 4.55 litres
   B. 7.50 litres
   C. 8.10 litres
   D. 9.55 litres

17. If the total surface area of a solid hemisphere is equal to its volume, find the radius.
   A. 3.0 cm
   B. 4.5 cm
   C. 5.0 cm
   D. 9.0 cm

18. Which of the following is true about parallelograms?
   A. Opposite angles are supplementary.
   B. Opposite angles are complementary.
   C. Opposite angles are equal.
   D. Opposite angles are reflex angles.

19. The diagram shows a circle with centre O. If $\angle STR = 29°$ and $\angle RST = 46°$, calculate the value of $\angle STO$.
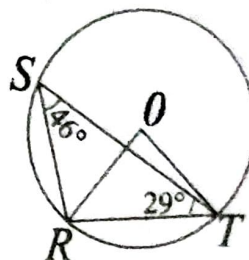
Not drawn to scale

   A. 12°
   B. 15°
   C. 29°
   D. 34°

20.
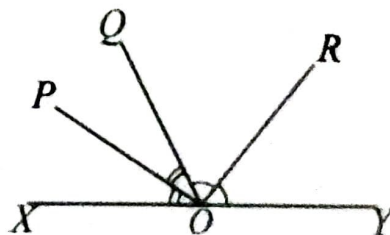
Not drawn to scale

In the diagram, XY is a straight line. $\angle POX = \angle POQ$ and $\angle ROY = \angle QOR$. Find the value of $\angle POQ + \angle ROY$.
   A. 60°
   B. 90°
   C. 100°
   D. 120°

June 2017 Objectives

21.

Not drawn to scale

The diagram shows a circle with centre O.
If $\angle ZYW = 33°$, find $\angle ZWX$.

A. 33°

B. 57°

C. 90°

D. 100°

22. In the diagram, PQ and PS are tangents to the circle centre O. If $\angle PSQ = m$, $\angle SPQ = n$ and $\angle SQR = 33°$, find the value of $(m + n)$.

Not drawn to scale

A. 103°

B. 123°

C. 133°

D. 143°

23. Calculate the gradient (slope) of the line joining points $(-1, 1)$ and $(2, -2)$

A. -1

B. $-\frac{1}{2}$
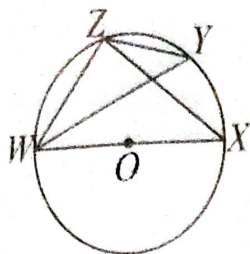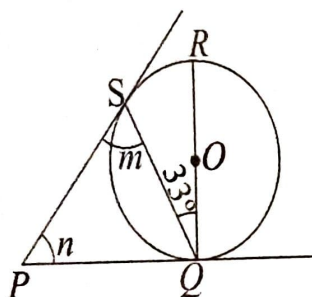
C. $\frac{1}{2}$

D. 1

24. If $P(2, 3)$ and $Q(2, -2)$ are points on a graph, calculate the length PQ.

A. 6 units

B. 5 units

C. 4 units

D. 2 units

25. A bearing of 320° expressed as a compass bearing is

A. N 50° W

B. N 40° W

C. N 50° E

D. N 40° E

26. Given that $\cos 30° = \sin 60° = \dfrac{\sqrt{3}}{2}$ and $\sin 30° = \cos 60° = \dfrac{1}{2}$, evaluate $\dfrac{\tan 60° - 1}{1 - \tan 30°}$.

A. $\sqrt{3} - 2$

B. $2 - \sqrt{3}$

C. $\sqrt{3}$

D. -2

27. A stationary boat is observed from a height of 100 m. If the horizontal distance between the observer and the boat is 80 m, calculate correct to two decimal places, the angle of depression of the boat from the point of observation.

A. 36.87°

B. 39.70°

C. 51.34°

D. 53.13°

28. The average age of a group of 25 girls is 10 years. If one girl, aged 12 years and 4 months joins the group, find, correct to **one** decimal place, the new average age of the group.

A. 10.1 years

B. 9.3 years

C. 8.7 years

D. 8.3 years

538

29.

KEY
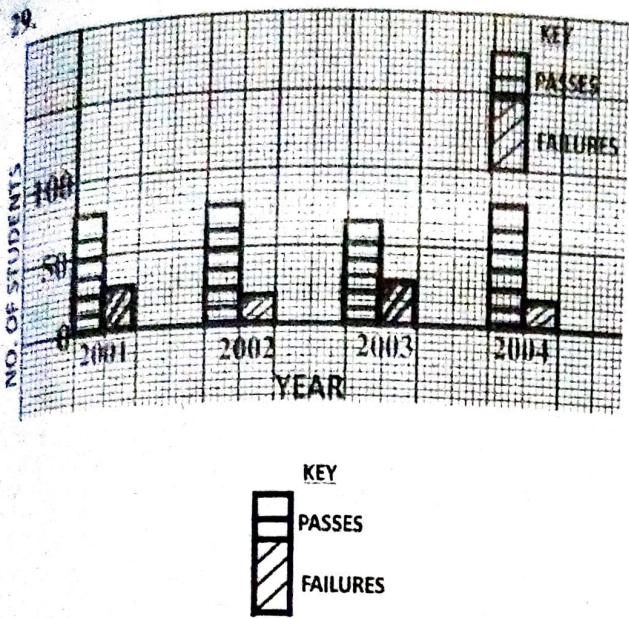
PASSES

FAILURES

The bar chart shows the statistics of the number of passes and failures in an examination in a school from 2001 to 2004. What is the ratio of the total number of passes to the total number of failures?

A. 60 : 13
B. 10 : 3
C. 5 : 1
D. 40 : 13

Marks	0	1	2	3	4	5
Frequency	7	4	18	12	8	11

The table gives the distribution of marks obtained by a number of pupils in a class test. Use this information to answer questions **30** and **31**.

30. Find the median of the distribution.
A. 4
B. 3
C. 2
D. 1

31. Find the **first** quartile.
A. 1.0
B. 1.5
C. 2.0
D. 2.5

32. In a class of 45 students, 28 offer Chemistry and 25 offers Biology. If each student offers **at least** one of the two

subjects, calculate the probability that a student selected at random from the class offers Chemistry **only**.

A. $\frac{2}{9}$
B. $\frac{4}{9}$
C. $\frac{5}{9}$
D. $\frac{7}{9}$

33. In what number base was the addition $1 + nn = 100$, where $n > 0$, done?
A. $n - 1$
B. n
C. $n + 1$
D. $n + 1$

34. Simplify $\sqrt{2}\left(\sqrt{6} + 2\sqrt{2}\right) - 2\sqrt{3}$.
A. 4
B. $\sqrt{3} + 4$
C. $4\sqrt{2}$
D. $4\sqrt{3} + 4$

35. Three exterior angles of a polygon are 30°, 40° and 60°. If the remaining exterior angles are 46° each, name the polygon.
A. Decagon
B. Nonagon
C. Octagon
D. Hexagon

36. In the diagram, $NQ//TS$, $\angle RTS = 50°$ and $\angle PRT = 100°$. Find the value of $\angle NPR$.
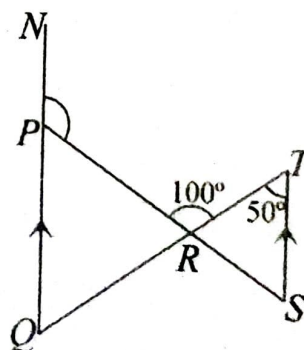
Not drawn to scale

A. 110°
B. 130°
C. 140°
D. 150°

37. Simplify the expression $\dfrac{a^2b^4 - b^2a^4}{ab(a + b)}$

A. $a^2 - b^2$
B. $b^2 - a^2$
C. $a^2b - ab^2$
D. $ab^2 - a^2b$

38. Find the 6th term of the sequence:

$$\frac{2}{3}, \frac{7}{15}, \frac{4}{15}, \ldots\ldots\ldots$$

A. $-\frac{1}{3}$

B. $-\frac{1}{5}$

C. $\frac{1}{15}$

D. $\frac{1}{5}$

39. The diagonal of a square is 60 cm. Calculate its perimeter.

A. $20\sqrt{2}$

B. $40\sqrt{2}$

C. $90\sqrt{2}$

D. $120\sqrt{2}$

40. The roots of a quadratic equation are $-\frac{1}{2}$ and $\frac{2}{3}$. Find the equation.

A. $6x^2 - x + 2 = 0$

B. $6x^2 - x - 2 = 0$

C. $6x^2 + x - 2 = 0$

D. $6x^2 + x + 2 = 0$

41. Make x the subject of the relation

$$d = \sqrt{\frac{6}{x} - \frac{y}{2}}$$

A. $x = \frac{6}{d^2} + \frac{12}{y}$

B. $x = \frac{12}{2d^2 - y}$

C. $x = \frac{12}{y} - 2d^2$

D. $x = \frac{12}{2d^2 + y}$

42. Consider the statements:

p: It is hot.

q: It is raining

Which of the following symbols correctly represents the statement?

"It is raining if and only if it is cold"?

A. $p \Leftrightarrow \sim q$

B. $q \Leftrightarrow p$

C. $\sim p \Leftrightarrow \sim q$

D. $q \Leftrightarrow \sim p$

43. Given that $t = 2^{-x}$, find 2^{x+1} in terms of t.

A. $\frac{2}{3}$.

B. $\frac{t}{2}$.

C. $\frac{1}{2t}$.

D. $2t$.

44.
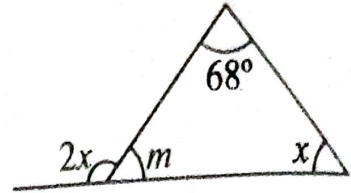
Not drawn to scale

Find the value of m in the diagram

A. 72°

B. 68°

C. 44°

D. 34°

45. Two bottles are drawn **with replacement** from a crate containing **8** Coke, **12** Fanta and **4** Sprite bottles. What is the probability that the **first** is Coke and the **second** is **not** Coke?

A. $\frac{1}{12}$

B. $\frac{1}{6}$

C. $\frac{2}{9}$

D. $\frac{3}{8}$

46. If the simple interest on a certain amount of money saved in a bank for 5 years at $2\frac{1}{2}\%$ per annum is ₦ 500.00, calculate the total amount due after 6 years at the same rate.

A. ₦ 2,500.00

B. ₦ 2,600.00

C. ₦ 4,500.00

D. ₦ 4,600.00

47. Calculate the variance of 2, 3, 3, 4, 5, 5, 5, 7, 7 and 9.

A. 2.2

B. 3.4

C. 4.0

D. 4.2

48. A circular pond of radius 4 m has a path of width 2.5 m round it. Find, correct to 2 decimal places, the area of the path. [Take $\pi = \frac{22}{7}$]

A. 7.83 m²

B. 32.29 m²

C. 50.29 m²

D. 82.50 m²

49. The graph of $y = ax^2 + bx + c$ is shown in the diagram. Find the minimum value of y.

A. –2.0   B. –2.1

C. –2.3   D. –2.5
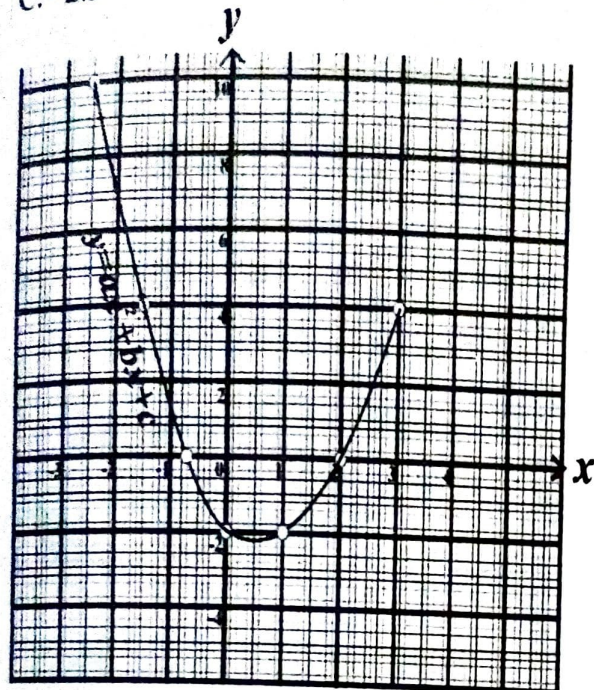
50.
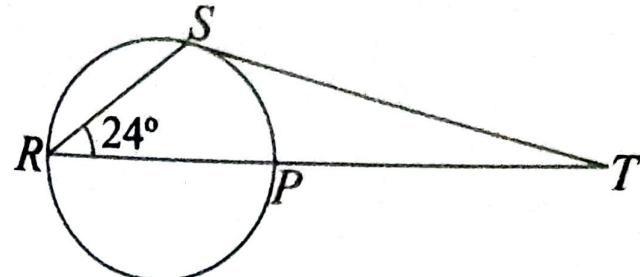
In the diagram, RP is a diameter of the circle RSP, RP is produced to T and TS is a tangent to the circle at S. If $\angle PRS = 24°$, calculate the value of $\angle STR$.

A. 24°   B. 42°

C. 48°   D. 66°

JUNE 2017 THEORY
QUESTIONS AND ANSWERS

Question 1

a) Given that $\log_{10} x = \overline{1}.3010$ and

$\log_{10} y = 1.6021$ find $\log_{10} \sqrt{\dfrac{x}{y}}$

b) A man bought some shirts for GH ¢720.00. If each shirt was GH ¢ 2.00 cheaper, he would have received 4 more shirts. Calculate the number of shirts bought.

Solution

a) $\log_{10}\sqrt{\dfrac{x}{y}} = \log_{10}\left(\dfrac{x}{y}\right)^{\frac{1}{2}} = \dfrac{1}{2}\log_{10}\left(\dfrac{x}{y}\right)$

$\therefore \log_{10}\sqrt{\dfrac{x}{y}} = \dfrac{1}{2}\left[\log_{10} x - \log_{10} y\right]$

But $\log_{10} x = \overline{1}.3010$ and $\log_{10} y = 1.6021$

$\log_{10}\sqrt{\dfrac{x}{y}} = \dfrac{1}{2}\left[\left(\overline{1}.3010\right) - \left(1.6021\right)\right]$

$\log_{10}\sqrt{\dfrac{x}{y}} = \overline{1}.6505 - 0.80105$

$\therefore \log_{10}\sqrt{\dfrac{x}{y}} = \overline{2}.8495 = -2 + 0.8495 = -1.1505$

Method 2

$\log_{10}\sqrt{\dfrac{x}{y}} = \dfrac{1}{2}\left[\left(\overline{1}.3010\right) - \left(1.6021\right)\right]$

$= \dfrac{1}{2}\left(\overline{3}.6989\right) = \dfrac{1}{2}\left(\overline{4} + 1.6989\right) = \left(\overline{2} + 0.8495\right)$

$\therefore \log_{10}\sqrt{\dfrac{x}{y}} = \overline{2}.8495 = -1.1505$

b) Let the number of shirts bought $= n$

Cost of n shirts $=$ GH ¢720.00

$\therefore$ Cost per shirt in cedis $= \dfrac{720}{n}$

If 4 more shirts had been received, then the total shirts would have been $= n + 4$
Hence if each shirt was GH ¢ 2.00 cheaper, we can obtain the following equation:

$\dfrac{720}{n} - 2 = \dfrac{720}{n+4}$

Expressing the left-hand-side as a common denominator gives:

$\dfrac{720 - 2n}{n} = \dfrac{720}{n+4}$

Cross-multiplying gives:
$(720 - 2n)(n + 4) = 720n$
Expanding gives:
$720n + 2,800 - 2n^2 - 8n = 720n$

$2,800 - 2n^2 - 8n = 0$

$2n^2 + 8n - 2,880 = 0$

$n^2 + 4n - 1,440 = 0$

$n^2 + 40n - 36n - 1,440 = 0$

$n(n + 40) - 36(n + 40) = 0$

$(n + 40)(n - 36) = 0$

Either $n = 36$ or $n = -40$
Since n cannot be negative, it implies that $n = 36$. Hence the number of shirts bought is 36.

Question 2

a) If $\sin 30° = \dfrac{1}{2}$, $\cos 45° = \dfrac{1}{\sqrt{2}}$ and

$\tan 60° = \sqrt{3}$, without using Mathematical tables or calculator, simplify

$\dfrac{\sin 30° + \cos 45°}{\tan 60°}$

b) Three interior angles of a polygon are 160° each. If the other interior angles are 120° each, find the number of sides of the polygon.

Solution

a) $\dfrac{\sin 30° + \cos 45°}{\tan 60°} = \dfrac{\frac{1}{2} + \frac{1}{\sqrt{2}}}{\sqrt{3}} = \dfrac{\frac{1}{2} + \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}}}{\sqrt{3}}$

$= \dfrac{\frac{1}{2} + \frac{\sqrt{2}}{2}}{\sqrt{3}} = \dfrac{\sqrt{3}\left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right)}{\sqrt{3}\sqrt{3}}$

$= \dfrac{\frac{\sqrt{3}}{2} + \frac{\sqrt{6}}{2}}{3} = \dfrac{\sqrt{3}}{6} + \dfrac{\sqrt{6}}{6} = \dfrac{\sqrt{3} + \sqrt{6}}{6}$

b) Let n = the number of sides of the polygon. The total for the three interior angles each having a value of $160°$ will be $3 \times 160°$

The number of interior angles remaining becomes: $n - 3$

Since the other interior angles are $120°$ each, the total for the other interior angles becomes: $(n-3) \times 120°$

Hence the sum of all the interior angles of the given polygon is:

$T = (3 \times 160°) + [(n-3) \times 120°] \cdots\cdots\cdots(1)$

Now the sum of the interior angles of a polygon is given by the expression:

$T = 180° \times (n-2) \cdots\cdots\cdots(2)$

From equations (1) and (2), we have:

$(3 \times 160°) + [(n-3) \times 120°] = 180° \times (n-2)$

$480° + 120°n - 360° = 180°n - 360°$

$480° + 120°n = 180°n$

$480° = 60°n$

$\therefore n = \dfrac{480°}{60°} = 8$

So the number of sides of the polygon is 8.

Question 3
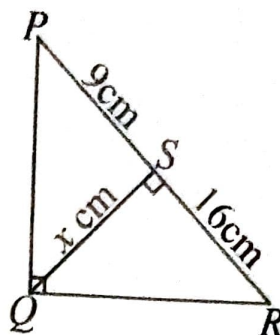
In the diagram, $\angle PQR = \angle PSQ = 90°$, $|PS| = 9$ cm, $|SR| = 16$ cm, and $|SQ| = x$ cm

Find:

a) the value of x.

b) $\angle QRS$, correct to the nearest degree.

c) $|PQ|$

Solution

a) **Finding x**

Let $\angle QRS = \theta$ as shown in the following diagram.
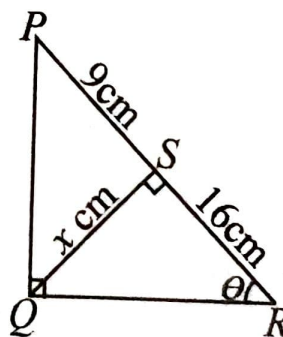
$\tan\theta = \dfrac{x}{16}$

Using the properties of similar triangles,

$\dfrac{x}{9} = \dfrac{16}{x}$

Cross-multiplying gives:

$x^2 = 144 \qquad\qquad \therefore x = \sqrt{144} = 12$ cm

b) Finding $\angle QRS$

$$\tan\theta = \frac{x}{16} = \frac{12}{16}$$

$$\theta = \tan^{-1}\left(\frac{12}{16}\right) = 36.87°$$

$\therefore \theta = 37°$ (correct to the nearest degree)

Hence $\angle QRS = 37°$

c) Finding the length of PQ
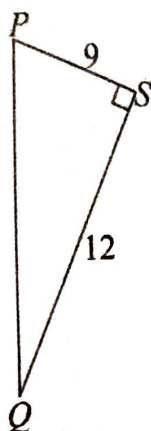
Applying Pythagoras theorem to the figure,

$$9^2 + 12^2 = |PQ|^2 \qquad 81 + 144 = |PQ|^2$$

$$|PQ| = \sqrt{81 + 144}$$

$$\therefore |PQ| = \sqrt{255} = 15 \text{ cm}$$

Question 4

a) A trader purchased 10 dozen eggs at ₦300.00 per dozen. On getting to his shop, he found that 20 eggs were broken. How much did he sell the remaining eggs if he made a profit of 10 %.

b) Thirty-five coloured balls were shared among four teams such that one team takes all the red balls. If the remainder is shared to the other teams in the ratio $4 : 3 : 2$ and the smallest share was 6 balls, how many red balls were there?

Solution

a) Cost price of eggs $= 300 \times 10 = ₦3,000.00$

Let $x =$ selling price of the remaining eggs

Profit = (Selling price) − (Cost price)

Profit $= x - 3,000$

$$\text{Profit percent} = \frac{\text{Profit}}{\text{Cost}} \times 100\ \%$$

$$\therefore \text{Profit percent} = \frac{x\text{-}3000}{3000} \times 100\ \%$$

but Profit percent = 10 %

$$10\% = \frac{x - 3000}{3000} \times 100\ \%$$

$$3000 \times 10 = (x\text{-}3000) \times 100$$

$$30 = x - 3000$$

$$\therefore x = 30 + 3000 = 3300$$

Hence selling price of the remaining eggs is ₦3,300.00

b) Let $r =$ Number of red balls

After all the red balls are taken by one team,

Number of balls remaining $= 35 - r$

Ratio for sharing remaining balls is 4:3:2

Total ratio is $4 + 3 + 2 = 9$

Total ratio of $9 \to 35 - r$ balls

$\therefore$ Smallest ratio of $2 \to \dfrac{2}{9} \times (35 - r)$ balls

But the smallest share was 6 balls

$$\therefore \frac{2}{9} \times (35 - r) = 6 \qquad\qquad 35 - r = 6 \times \frac{9}{2}$$

$$35 - r = 27 \qquad\qquad \therefore r = 8$$

So there were 8 red balls.

Question 5

a) The probabilities that Mensah will pass a Mathematics and an Economics tests are $\frac{3}{4}$ and $\frac{5}{8}$ respectively. If the probability that he passes **at least** one of the subjects is $\frac{7}{12}$, what is the probability that he passes **both** subjects?

b) In a class of 30 students, 25 offer Biology, 21 offer Physics and **each** student offers **at** least one of the subjects. If a prefect is selected from the class, what is the probability that she offers one subject only.

Solution

a) Let $P(M)$ = Probability that he passes Math

$P(E)$ = Probability that he passes economics

$P(M) = \frac{3}{4}$; $P(E) = \frac{5}{8}$; $P(E \cup M) = \frac{7}{12}$;

$P(E \cup M) = P(E) + P(M) - P(E \cap M)$
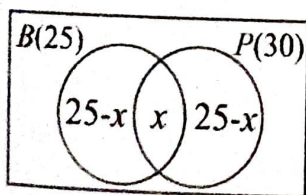
$\therefore P(E \cap M) = P(E) + P(M) - P(E \cup M)$

$P(E \cap M) = \frac{3}{4} + \frac{5}{8} - \frac{7}{12}$

$P(E \cap M) = \frac{18 + 15 - 14}{24}$

Hence $P(E \cap M) = \frac{19}{24}$

The probability that he passes both subjects $= \frac{19}{24}$

b) Let U = {students in class}

B = {students who offer Biology}

P = {students who offer Physics}

x = number offering both one subject

The Venn diagram is as follows:

$n(U) = 30$

$(25 - x) + x + (21 - x) = 30$

$46 - x = 30$

$\therefore x = 16$

Number that offered only one subject

$= (25 - x) + (21 - x)$

$= (25 - 16) + (21 - 16)$

$= 14$

Probability that she offers one subject only

$= \frac{\text{Number that offered only one subject}}{\text{Number of students in class}}$

$= \frac{14}{30} = \frac{7}{15} = 0.4667$

Question 6

A publisher prints 30,000 copies of a book at GH₵2.00 **each** and sold them for GH₵ 2.76 **each**. The publisher agrees to pay 10 % of the selling price for the first 6,000 copies sold and $12\frac{1}{2}$ % of the selling price for all copies sold in excess of 6,000. If 25,380 copies of the book were sold,

a) calculate, correct to the nearest Ghana Cedi, the:

   i) total amount received by the author;

   ii) net profit the publisher makes after he has paid the author.

b) find, correct to **one** decimal place, the publisher's net profit as a percentage of the author's total receipt.

Solution

a)   Royalties for the first 6,000 copies

$= \frac{10}{100} \times 2.76 \times 6,000$

$= $ GH₵1,656.00

Now the remaining copies $= 25,380 - 6,000$

$\therefore$ The remaining copies $= 19,380$

Royalties for the remaining copies

$= \frac{12\frac{1}{2}}{100} \times 2.76 \times 19,380$

$= \frac{25}{200} \times 2.76 \times 19,380$

$= $ GH₵6,686.10

i) Total amount received by the author

$= $ (Royalties for the first 6,000 copies) + (Royalties for the remaining copies)

$= $ GH₵ 1,656.00 + GH₵ 6,686.10

$= $ GH₵ 8,342.10

ii) Printing cost
= (Number of copies of the book) ×
(Cost of each book)
= 30,000 × GH₵2.00 = GH₵ 60,000

Amount received after selling
25.380 copies = 2.76 × 25,380

= GH¢70,048.80

Total expenditure incurred by publisher
= Printing cost + Amount paid to the author
= 60,000 + 8,342.10
= GH¢68,342.10

ii) Net profit the publisher makes after paying
the author
= (Amount received) – (Total expenditure)
= 70,048.80 – 68342.10 = GH¢1,706.70

b) The publisher's net profit as a percentage of
the author's total profit

$$= \frac{\text{Net profit}}{\text{Author's receipt}} \times 100\%$$

$$= \frac{1,706.70}{8342} \times 100 = 20.5\%$$

Question 7
A bag of food aid is released from an
aeroplane when it is 1000 m above a military
camp. The height h metres, of the bag above
the camp at time t seconds is given by the
relation $h = 100 - 5t^2$.
a) Copy and complete the following table for
the relation $h = 100 - 5t^2$.

t(s)	0	1	3	5	7	9	11	13	15
h(m)				875			395		

b) Using a scale of 2 cm to 2 seconds on the
t – axis and 2 cm to 1000 m on the h – axis,
draw a graph of the relation $h = 100 - 5t^2$
for $0 \le t \le 15$.

c) Use the graph to find, correct to one decimal
place, the:
i) time the bag takes to reach the ground.
ii) time the bag takes to drop through the
first 650 m.
iii) height of the bag above the camp after
falling for 7.5 seconds.

Solution
a) $h = 100 - 5t^2$

When $t = 0$, $h = 1000 - 5 \times 0^2 = 1000$

When $t = 1$, $h = 1000 - 5 \times 1^2 = 995$

When $t = 3$, $h = 1000 - 5 \times 3^2 = 955$
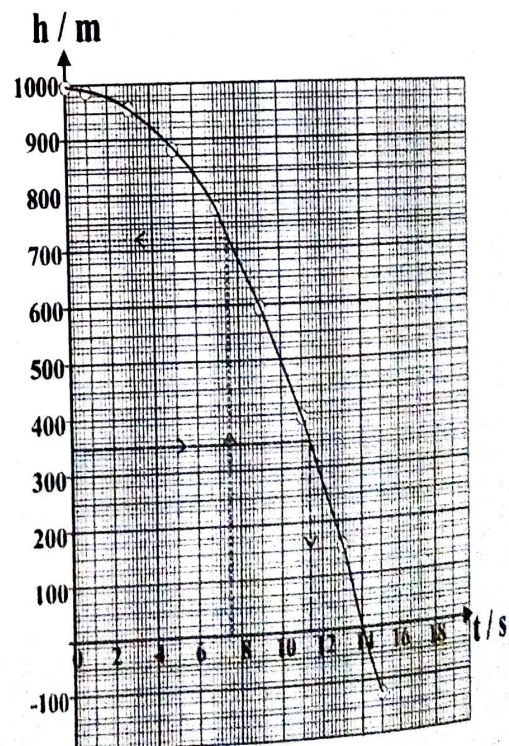
When $t = 7$, $h = 1000 - 5 \times 7^2 = 755$

When $t = 9$, $h = 1000 - 5 \times 9^2 = 595$

When $t = 13$, $h = 1000 - 5 \times 13^2 = 155$

The complete table is as follows:

t (s)	0	1	3	5	7	9	11	13	15
h(m)	1000	995	955	875	755	595	395	155	125

b) The following is the graph:

h / m

c) i) The time the bag takes to reach the ground can be obtained by locating where the curve cuts the horizontal axis. From the graph, the required time is **14.2** seconds.

ii) 1000 m – 650 m = 350 m
So from the graph, the required time it takes the bag to drop through the first 650 m is **11.4** seconds.

iii) From the graph, the height of the bag above the camp after falling for 7.5 seconds is **720 m**.

Question 8

a)
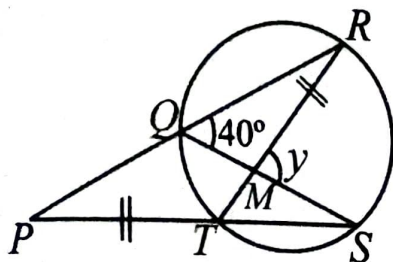
Not drawn to scale

In the diagram, $\angle RQS = 40°$, $|RT| = |PT|$ and $\angle RMS = y$. Find the value of y.

b) XY is a tangent to a circle LMN at the point M. XLN is a straight line. $\angle NXM = 34°$ and $\angle NMY = 65°$.

i) Illustrate the information in a diagram.
ii) Find the value of:
   α) $\angle MLX$;
   β) $\angle LNM$.

Solution

$\angle RTS = \angle RQS = 40°$ because the angles made by chord RS on the circumference are the same.

$\angle RTS = \angle RPT + \angle PRT$

But $\angle RPT = \angle PRT$

$\Rightarrow \angle RTS = \angle PRT + \angle PRT \Rightarrow \angle RTS = 2 \times \angle PRT$

$\therefore \angle PRT = \dfrac{\angle RTS}{2} = \dfrac{40°}{2} = 20°$

Consider $\triangle MQR$ as follows:
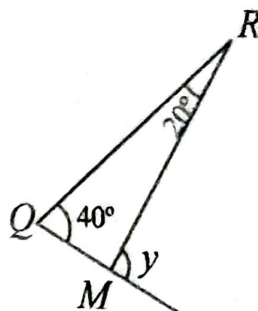
The exterior angle of $\triangle MQR$ is:
$y = 40° + 20°$ $\therefore y = 60°$
(**Note** that $\angle MRQ$ is the same as $\angle PRT$)

b) i) The following is the illustration of the given information:
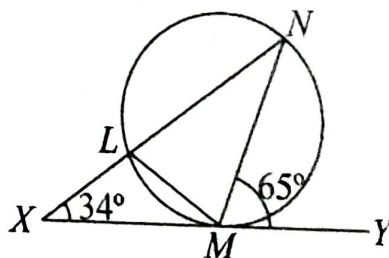
ii) α) Finding $\angle MLX$
$\angle MLN = \angle NMY$ (i.e angles in alternate segments)
$\therefore \angle MLN = 65°$
$\angle MLN + \angle MLX = 180°$ (angles on a straight line)
$\therefore \angle MLX = 180° - \angle MLN = 180° - 65° = 115°$

β) Finding $\angle LNM$
Method 1
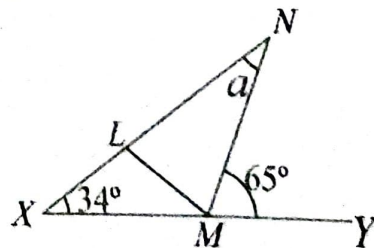
June 2017 Theory

The exterior angle of $\triangle XMN$ is given by

$65° = 34° + a \qquad \Rightarrow a = 65° - 34° = 31°$

$\therefore \angle LNM = 31°$

Method 2

$\angle XMN = 180° - \angle NMY = 180° - 65° = 115°$

But $\angle XMN + \angle NXM + \angle XNM = 180°$

$\Rightarrow 115° + 34° + \angle XNM = 180°$

$\Rightarrow \angle XNM = 180° - 115° - 34° = 31°$

$\therefore \angle LNM = 31°$ since it is the same as $\angle XNM$

Question 9

a) If $T = WP[M^2 - (M - S)^2]$, express M in terms of T, W, P and S.

b) A point X is between two towers TP and QW and are all on the same horizontal ground. The angles of elevation of the tops T and Q from X are $62°$ and $48°$ respectively. $TP = 100$ m and $PW = 80$ m.

i) Illustrate the information in a diagram.

ii) Calculate, correct to the **nearest** metre, $|QW|$.

Solution

a) If $T = WP[M^2 - (M - S)^2]$

$T = WP\left[M^2 - \left(M^2 - 2MS + S^2 \right) \right]$

$T = WP\left[M^2 - M^2 + 2MS - S^2 \right]$

$T = WP\left(2MS - S^2 \right) \Rightarrow T = 2WPMS - WPS^2$

$2WPMS = T + WPS^2 \qquad \therefore M = \dfrac{T + WPS^2}{2WPS}$

b) i) The illustration of the information is as follows:
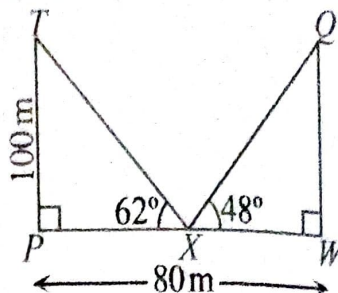
ii) $\tan 62° = \dfrac{100}{|PX|} \Rightarrow |PX| = \dfrac{100}{\tan 62°} = \dfrac{100}{1.8807}$

$\therefore |PX| = 53.17$ m

$|XW| = 80 - |PX| = 80 - 53.17 = 26.83$ m

$\tan 48° = \dfrac{|QW|}{26.83} \Rightarrow |QW| = 26.83 \times \tan 48°$

$\therefore |QW| = 29.80$ m ≈ 30 m (nearest metre)

Question 10

a) If $(x - 1)\log_{10} 4 = x\log_{10} 16$, without using Mathematical tables or calculator, find the value of x.

b)
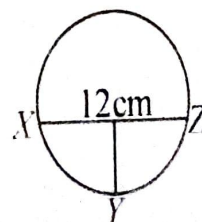
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In the diagram, XZ is a chord which is 12 cm long. If the perpendicular distance from the midpoint of the chord to a point Y on the circumference of the circle is 4 cm, calculate, correct to **one** decimal place, the perimeter of the sector $OXZY$.

[Take $\pi = \frac{22}{7}$]

Solution

a) $(x-1)\log_{10} 4 = x\log_{10} 16$

$\log_{10} 4^{(x-1)} = \log_{10} 16^x \qquad \Rightarrow 4^{(x-1)} = 16^x$

$4^{(x-1)} = 4^{2x} \qquad\qquad\qquad \Rightarrow (x-1) = 2x$

$-1 = 2x - x \qquad\qquad\qquad \therefore x = -1$

b) Let r = radius of the circle

x = distance from O to the centre of the circle
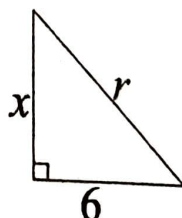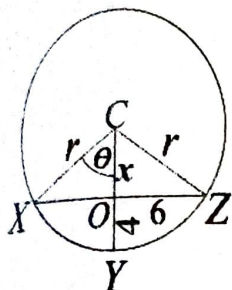
Using Pythagoras theorem, $x^2 + 6^2 = r^2$ ––(1)

but $r = x + 4$ –––––––––––––(2)

Substituting equation (2) into equation (1) gives:

$x^2 + 6^2 = (x+4)^2$

$x^2 + 6^2 = x^2 + 8x + 16 \quad \Rightarrow 8x = 20$

$\therefore x = \dfrac{20}{8} = 2.5$

From equation (2),

$r = x + 4 = 2.5 + 4 = 6.5$ cm
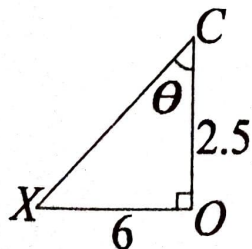
$\tan\theta = \dfrac{6}{2.5} \qquad \therefore \theta = \tan^{-1}\left(\dfrac{6}{2.5}\right) = 67.38°$

Angle subtended at the centre of the circle

$= 2\theta = 2 \times 67.38° = 134.76°$

Length of arc $XZ = \dfrac{134.76°}{360°} \times 2\pi r$

$\Rightarrow$ Length of arc $XZ = \dfrac{134.76°}{360°} \times 2 \times \dfrac{22}{7} \times 6.5$

$\therefore$ Length of arc $XZ = 15.2942$ cm
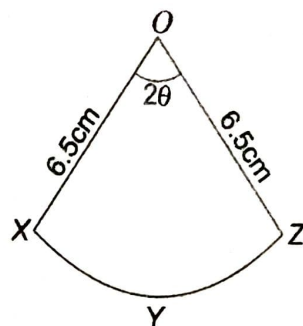
Using the figure,

Perimeter of the sector $OXZY$,

= 6.5 cm + 6.5 cm + 15.2942 cm

= 28.2942 cm ≈ 28.3 cm (1 d.p)

Question 11

The distribution of marks scored by some students in a test is as follows:

Marks	1	2	3	4	5
Number of students	$p+2$	$p-1$	$2p-3$	$p+4$	$3p+4$

a) If the mean mark is $3\frac{5}{22}$, find the value of p.

b) Find the:

   i) interquartile range.

   ii) probability of selecting a student who scored **at least** 4 marks in the test.

Solution

x	1	2	3	4	5	
Number of students f	$p+2$	$p-1$	$2p-3$	$p+4$	$3p+4$	$\Sigma f = 8p-2$
fx	$p+2$	$2p-2$	$6p-9$	$4p+16$	$15p-20$	$\Sigma fx = 28p-13$

a) Mean $= \dfrac{\Sigma fx}{\Sigma f} = \dfrac{28p-13}{8p-2} = 3\frac{5}{22}$ $\therefore \dfrac{28p-13}{8p-2} = \dfrac{71}{22}$

Cross-multiplying gives: $22(28p-13) = 71(8p-2)$

$\Rightarrow 616p - 286 = 568p - 142$ $\Rightarrow 616p - 568p = -142 + 286$

$\Rightarrow 48p = 144$ $\therefore p = 3$

b) i) In order to find the interquartile range, we can re-construct the given table as follows:

x	1	2	3	4	5	
f	5	2	3	7	5	$\Sigma f = 22$

$\Sigma f = 8p - 2 = 8 \times 3 - 2 = 22$

Lower quartile $= \dfrac{\Sigma f}{4} = \dfrac{22}{4} = 5.5^{th}$ observation

By inspection using the table, the 5.5^{th} observation corresponds to 2. Hence the lower quartile is 2.

Upper quartile $= \dfrac{3 \times \Sigma f}{4} = \dfrac{3 \times 22}{4}$

$= 16.5^{th}$ observation

By inspection using the table, the 16.5^{th} observation corresponds to 4. Hence the lower quartile is 4.

Inter-quartile range
$=$ (Upper quartile) $-$ (Lower quartile)
$= 4 - 2 = 2$

ii) Students who scored at least 4 marks
$= 7 + 5 = 12$
$\therefore$ P(selecting a student who scored at least 4)

$= \dfrac{12}{22} = \dfrac{6}{11} = 0.545$

Question 12

a) The operation $*$ is defined on the set of real numbers, **R** by: $x*y = \dfrac{x+y}{2}$, $x, y \in \mathbf{R}$.

i) Evaluate $3 * \frac{2}{5}$.

ii) If $8 * y = 8\frac{1}{4}$, find the value of y.

b) In $\triangle ABC$, $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$.

If **P** is the midpoint of $\overrightarrow{AB}$, express $\overrightarrow{CP}$ as a column vector.

Solution

a) i) $x*y = \dfrac{x+y}{2}$ $\Rightarrow 3*\frac{2}{5} = \dfrac{3+\frac{2}{5}}{2}$

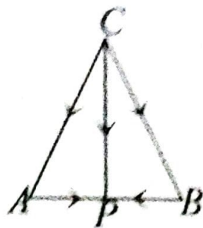$\therefore 3*\frac{2}{5} = \dfrac{\frac{15+2}{5}}{2} = \dfrac{\frac{17}{5}}{2} = \dfrac{17}{10}$

a) $x + y = 8\frac{1}{2} \Rightarrow \frac{x+y}{2} = 8\frac{1}{4}$

$\Rightarrow \frac{x+y}{2} = \frac{33}{4} \Rightarrow 4(8 + y) = 2(33)$

$\Rightarrow 32 + 4y = 66 \Rightarrow 4y = 66 - 32$

$\Rightarrow 4y = 34 \qquad \therefore y = \frac{34}{4} = 8\frac{1}{2} = 8.5$

b) The triangle can be illustrated as follows:

From the diagram,

$$\vec{CP} = \vec{CA} + \vec{AP} \text{ ---------------(1)}$$

but $\vec{CA} = -\vec{AC} = -\begin{pmatrix} 3 \\ -8 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$

Also, $\vec{AP} = \frac{1}{2}\vec{AB} = \frac{1}{2}\begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

From equation (1),

$\vec{CP} = \vec{CA} + \vec{AP} = \begin{pmatrix} -3 \\ 8 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3-2 \\ 8+3 \end{pmatrix}$

$\therefore \vec{CP} = \begin{pmatrix} -5 \\ 11 \end{pmatrix}$

Question 13

a) Using completing the square method, solve, correct to **two** decimal places, the equation $3y^2 - 5y + 2 = 0$.

b) Given that $M = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$, $N = \begin{pmatrix} m & x \\ n & y \end{pmatrix}$ and

$MN = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$, find the matrix **N**.

Solution

a) $3y^2 - 5y + 2 = 0 \Rightarrow 3y^2 - 5y = -2$

In order to complete the squares, we may divide through by 3 (i.e the coefficient of y) as follows:

$\frac{3y^2}{3} - \frac{5y}{3} = \frac{-2}{3} \qquad \Rightarrow y^2 + \left(\frac{-5}{3}\right)y = \frac{-2}{3}$

We can now add the square of **half** of the coefficient of y to both sides of the equation as follows:

$\Rightarrow y^2 + \left(\frac{-5}{3}\right)y + \left(\frac{-5}{6}\right)^2 = \frac{-2}{3} + \left(\frac{-5}{6}\right)^2$

$\Rightarrow y^2 - \frac{5}{3}y + \left(\frac{-5}{6}\right)^2 = \frac{1}{36}$

Factorizing this result gives the following:

$\left(y - \frac{5}{6}\right)^2 = \frac{1}{36} \qquad \Rightarrow y - \frac{5}{6} = \pm\sqrt{\frac{1}{36}}$

$\Rightarrow y = \frac{5}{6} \pm \sqrt{\frac{1}{36}} \qquad \Rightarrow y = \frac{5}{6} \pm \frac{1}{6}$

Hence either $y = \frac{5}{6} + \frac{1}{6} = 1$

Or $y = \frac{5}{6} - \frac{1}{6} = \frac{2}{3} = 0.67$

$\therefore \{y : y = 1, \frac{2}{3}\}$

b) $M = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$, $N = \begin{pmatrix} m & x \\ n & y \end{pmatrix}$

$MN = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} m & x \\ n & y \end{pmatrix}$

$\Rightarrow MN = \begin{pmatrix} m+2n & x+2y \\ 4m+3n & 4x+3y \end{pmatrix}$

But $MN = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$

$$\therefore \begin{pmatrix} m+2n & x+2y \\ 4m+3n & 4x+3y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

We can now equate the corresponding components to obtain the following equations:

$m+2n=2$ ----------------------------(1)

$4m+3n=3$ ---------------------------(2)

$x+2y=1$ ----------------------------(3)

$4x+3y=4$ ---------------------------(4)

(1) $\times 3$: $3m+6n=6$ -------------- (5)

(2) $\times 2$: $8m+6n=6$ -------------- (6)

(6) $-$ (5): $5m=0$ $\qquad \Rightarrow m=0$

Put $m=0$ into (2) gives: $0+2n=2$ $\quad \therefore n=1$

(3) $\times 3$: $3x+6y=3$-------------- (7)

(4) $\times 2$: $8x+6y=8$-------------- (8)

(8) $-$ (7): $5x=5$ $\qquad \Rightarrow x=1$

Put $x=1$ into (3) gives: $1+2y=1$ $\quad \therefore y=0$

We can now put the values of $m=0$, $n=1$,

$x=1$ and $y=0$ into the matrix $N=\begin{pmatrix} m & x \\ n & y \end{pmatrix}$

Hence $N=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

40 NOV. 2017 OBJECTIVE TEST

1. If $\sqrt{75} - \sqrt{27} + \sqrt{48} = p\sqrt{3}$, find the value of p.
   A. 4
   B. 5
   C. 6
   D. 7

2. Correct 4897 to three significant figures.
   A. 4900
   C. 4898
   B. 4899
   D. 4800

3. The actual mass of a textbook is 75.6 g. A student measured the mass as 77.0 g. Find, correct to one decimal place, the percentage error in the measurement.
   A. 1.4 %
   B. 1.9 %
   C. 2.5 %
   D. 2.9 %

4. Kwame borrowed GH¢ 300.00. He repaid the loan with six equal installments of GH¢55.00. How much interest did he pay?
   A. GH¢ 3.00
   B. GH¢ 30.00
   C. GH¢33.00
   D. GH¢ 330.00

5. Which of the following statements is a proposition?
   A. Good morning sir
   B. Is she okay?
   C. Oh my God
   D. It is raining

6. Write down the 15th term of the sequence 1, 2, 3; 2, 3, 4; 3, 4, 5;...
   A. 13, 14, 15
   B. 14, 15, 16
   C. 15, 16, 17
   D. 16, 17, 18

7. If $243_n + 452_n = 1135_n$, find the value of n.
   A. 6
   B. 7
   C. 8
   D. 9

8. Given that $a = 3$ and $b = \frac{5}{3}$, find the value of $3a^2b - 9ab^2$

A. 30
C. –17
B. 25
D. –30

9. Find r in terms of q, s and t in the equation:
   $\frac{1}{r} - \frac{2q}{s} = t$.
   A. $r = \frac{s-t}{2q}$
   B. $r = \frac{s+t}{2q}$
   C. $r = \frac{s}{st-2q}$
   D. $r = \frac{s}{st+2q}$

10. If $(x - y)$ is one of the factors of $xy - ys - y^2 + xs$, find the other factor.
    A. $(y - s)$
    B. $(s - y)$
    C. $(y + s)$
    D. $(s - x)$

11. If $\frac{x}{x-z} = \frac{y}{z-y}$, find z when $x = 2$ and $y = 3$
    A. $\frac{2}{5}$
    B. $\frac{2}{3}$
    C. $\frac{7}{3}$
    D. $\frac{12}{3}$

12. Given that $x \in \{integers\}$, find the solution set of the inequality: $5x + 4 \geq 2x + 10$
    A. $\{.....,-4,-3,-2\}$
    B. $\{1,2,3,.....\}$
    C. $\{2\}$
    D. $\{2,3,4,....\}$
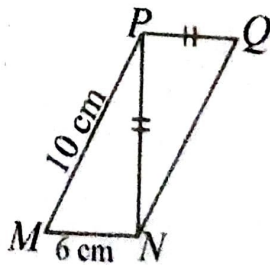
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13. In the diagram, $|PM| = 10$ cm, $|MN| = 6$ cm and $\angle MNP = \angle NPQ = 90°$. Calculate $|NQ|$

A. $6\sqrt{2}$ cm

B 10 cm

C. $8\sqrt{2}$ cm

D. $10\sqrt{2}$ cm

14. An empty container with rectangular base is 50 cm long and 24 cm wide. If 36 liters of water is poured into the container, how high would the water level rise?

A. 32 cm

B. 30 cm

C. 10 cm

D.3 cm

15. The diameter of the base of a cone is 8 cm and its height is 3 cm. Calculate the curved surface area of the cone. [Take $\pi = \frac{22}{7}$]

A. $62.86\,cm^2$

B. $75.43\,cm^2$

C. $125.71\ cm^2$

D. $201.10\ \ cm^2$

16. The interior angle of a regular polygon of *n* sides is 120° what is the value of *n*?

A.36

B. 24

C.12

D. 6

17. In the figure, $MO // PQ, |NP| = |PO|,$

$\angle MNP = 4a^o$ and $\angle QPO = a^o$ find the value of a.

A. 36

B. 30

C. 25

D. 20
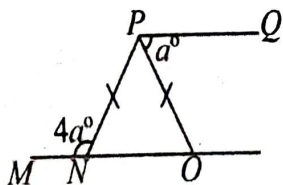
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18. In the figure, *PQ* is a tangent to the circle *LMR* at *R*. if *LM* is a diameter and <*RLM* = 35°, calculate the value of *x*

A. 75°   B. 65°   C. 55°   D. 45°
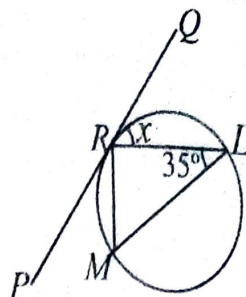
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19. In the diagram, *O* is the centre of the circle, *POR* is a diameter and $\angle PQS = 37°$ What is $\angle SPR$?
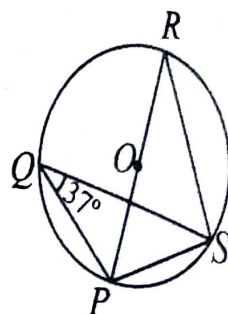
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A. 147°

B. 127°

C. 55°

D. 37°

20. In the diagram, <*PQT*= 150°, |*QT*| = |*QS*|, *PQ* // *UT*, *QR* // *TS* and ∠*SQT* is a right angle. Find the value of <*PQR*.
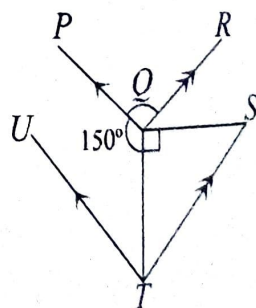
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A. 35°

B. 75°

C. 105°

D. 150°

21. The gradient of a straight line which passes through the point $(-2, 3)$ is 2. Find its equation.
    A. $y = 2x - 7$
    B. $y = 2x - 1$
    C. $y = 2x + 1$
    D. $y = 2x + 7$

22. In the diagram $\triangle OPQ$ is drawn in the Cartesian plane $(x - y$ plane$)$.
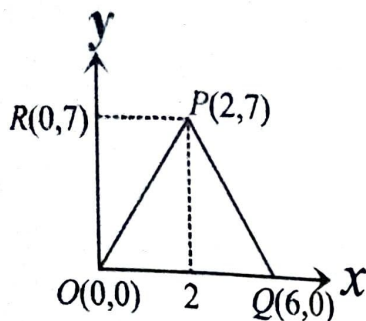
Find, in square units, the area of the triangle
    A. 6
    B. 12
    C. 14
    D. 21

23. Evaluate $\sin 210° + \cos 210°$
    A. $\dfrac{1 + \sqrt{3}}{2}$
    B. $\dfrac{\sqrt{2}}{2}$
    C. $-\dfrac{\sqrt{3}}{2}$
    D. $-\dfrac{1 - \sqrt{3}}{2}$

24.
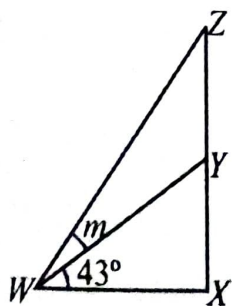
In the diagram, Y is the midpoint of XZ, $\angle YWX = 43°$ and $\angle YWZ = m$. Find, correct to one decimal place, the value of m.
    A. $18.8°$
    B. $23.8°$
    C. $47.0°$
    D. $61.8°$

25. From the top of a cliff 90 m high, the angle of depression of a boat on the sea is $36.2°$. Calculate, correct to one decimal place, the distance of the boat from the cliff.
    A. 65.9m
    B. 111.5m
    C. 123.0m
    D. 132.4m

26. The pie chart shows the distribution of the means by which students travelled to school at the beginning of the term. What is the angle of the sector that represents the students who travelled by motor-cycle?
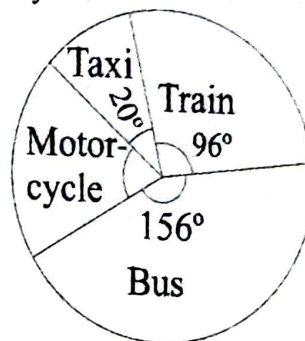
    A. $63°$
    B. $72°$
    C. $78°$
    D. $99°$

27. A fair die is thrown **once**. Find the probability of obtaining a 3, 4 **or** 5
    A. $\dfrac{1}{216}$
    B. $\dfrac{1}{3}$
    C. $\dfrac{1}{2}$
    D. $\dfrac{2}{3}$

28. What is the upper quartile of the following distribution?
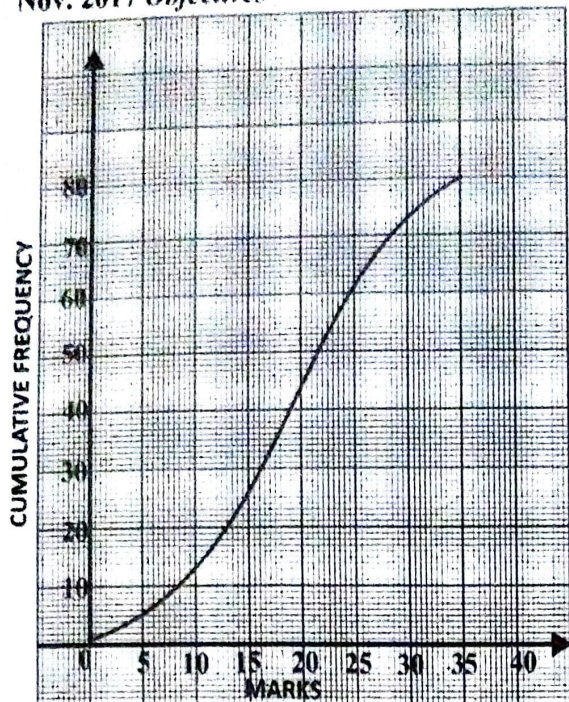
A. 14   B. 20
C. 25   D. 60

29. The mean of ten numbers is 9.5. If the mean of eight of them is 8.2, find the mean of the remaining two.
    A. 1.3   B. 2.9
    C. 14.2   D. 14.7

30. Given that m varies inversely as the cube root of n and $n = 64$ when $m = 3$, find m when $n = 8$.
    A. 6   B. 9
    C. 36   D. 42

31. If $X = \{1, 3, 5, 2\}$ and $Y = \{1, 2, 6, 7, 8\}$ are subsets of the universal set $U = \{1, 2, 3, ..., 10\}$, find the number of elements in $(X \cup Y)$.
    A. 3   B. 4
    C. 5   D.6

32. Find the value of $8 + 6(\text{mod } 5)$
    A. 1   B. 2
    C. 3   D. 4

33. Solve $2y^2 - y - 1 = 0$.
    A. 2, -1   B. $-1, \dfrac{1}{2}$
    C. $-1, -\dfrac{1}{2}$ D. $1, -\dfrac{1}{2}$

Use the graph to answer question 34.
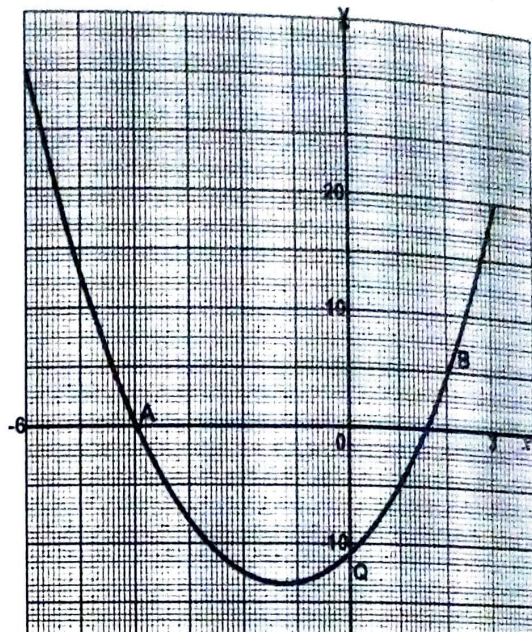
34. Find the equation of the graph.
    A. $y = 2x^2 + x - 12$
    B. $y = 2x^2 + 5x - 12$
    C. $y = 2x^2 + 6x - 9$
    D. $y = 2x^2 + 7x - 12$

35. Evaluate: $\dfrac{\log \sqrt{2}43 - \log \sqrt{27}}{\log 81}$.
    A. $\dfrac{1}{4}$ B. $\dfrac{3}{8}$
    C. $\dfrac{3}{4}$ D. $1\dfrac{1}{2}$

36. The sum and ratio of two numbers are 84 and $\dfrac{5}{7}$ respectively. By how much is the larger number greater than the smaller number?
    A. 12   B. 13   C.14   D. 15

556

37. Factorise: $3ax + y - 3x - ay$
    A. $(a + 1)(3x - y)$ B. $(a - 1)(3x - y)$
    C. $(a - 1)(3x + y)$ D. $(a + 1)(3x + y)$

38. Two parallel chords, with lengths 12 cm and 16 cm, lie on opposite sides of the centre of a circle of radius 10 cm. Calculate the distance between the chords
    A. 14 cm   B. 20 cm
    C. 28 cm   D. 48 cm

39.
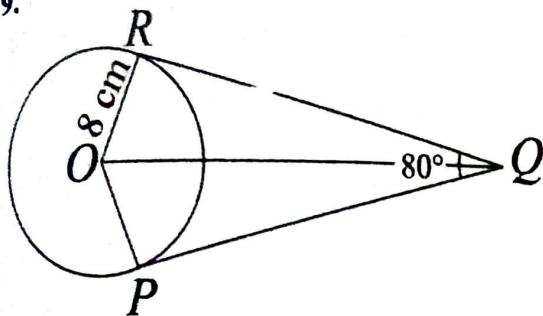
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In the diagram, RQ and PQ are tangents to the circle centre O and $\angle PQR = 80°$. Calculate the reflex $\angle POR$.
    A. $160°$ B. $200°$
    C. $260°$ D. $310°$

40.
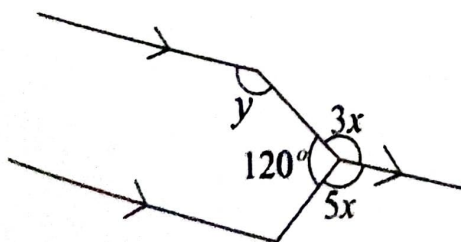
Find the value of the angle marked y in the diagram.
    A. $120°$ B. $90°$
    C. $60°$ D. $30°$

41. The table shows the distribution of the height of plants in a nursery.

Height	2	3	4	5	6
Frequency	2	4	5	3	1

Find the percentage of plants with heights above the modal height.
    A. 20% B. $26\frac{2}{3}\%$
    C. $33\frac{1}{3}\%$ D. 40%

42. The base of a pyramid is a square. If the height and volume are 9 cm and 192 cm^3 respectively, find the length of the base.
    A. 9 cm   B. 8 cm
    C. 7 cm   D. 6 cm

43. The standard deviation of a given set of numbers is 15. What is the variance of the numbers?
    A. 3.87   B. 30
    C. 205   D. 225

44. The probability that a seed planted on a piece of farmland will germinate is $\frac{2}{5}$. Find the probability that if 3 seeds are planted on the farmland, none of them will germinate.
    A. $\frac{8}{125}$ B. $\frac{9}{125}$
    C. $\frac{27}{125}$ D. $\frac{3}{5}$

45. If $(0.25)^x = 32$, find the value of x
    A. $\frac{5}{2}$ B. $\frac{3}{2}$
    C. $-\frac{3}{2}$ D. $-\frac{5}{2}$

46. Given that ₦ 49,500.00 was shared in the ratio $\frac{1}{2}:\frac{1}{4}:\frac{1}{6}$ how much was the smallest share?

A. ₦ 8, 250.00   B. ₦ 9, 000.00

C. ₦ 24, 750.00   D. ₦ 27, 000.00
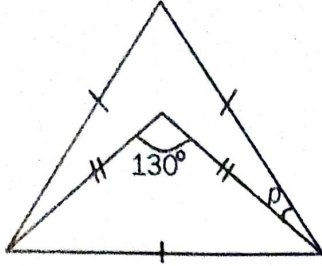
47. Calculate the value of the angle marked p in the diagram.

A. 15°   B. 30°

C. 35°   D. 40°

48. *PQRS* is a trapezium such that

$|PQ| = 10$ cm, $|PS| = 16$cm and

$|SR| = 16$ cm. If QR is perpendicular to

PQ, calculate the area of the trapezium.

A. 78 cm^2 B. 80 cm^2

C. 104 cm^2 D. 130 cm^2

49. Find the **least** value of x for which the expression $\dfrac{2-x-x^2}{1+x}$ equals zero.

A. 2   B. 1   C. -1   D. -2

50.
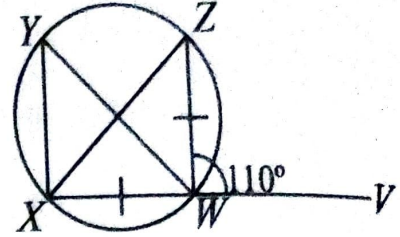
If $\angle VWX = 110°$, find $\angle WXZ$

A. 110°   B. 100°

C. 85°   D. 55°

NOV. 2017 THEORY
QUESTIONS AND ANSWERS

Question 1

a) Simplify: $\dfrac{2+\sqrt{3}}{\sqrt{3}} - \dfrac{\sqrt{2}-2}{2}$

b) Mr. Kofi sold a machine and made a profit of 15 %. The buyer later sold it to Mr. Nana at a loss of 10 %. If Mr. Nana paid GH¢20,700.00 for the machine, how much did Mr. Kofi buy it?

Solution

a) $\dfrac{2+\sqrt{3}}{\sqrt{3}} - \dfrac{\sqrt{2}-2}{\sqrt{2}} =$

$\dfrac{2+\sqrt{3}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} - \dfrac{\sqrt{2}-2}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{2\sqrt{3}+3}{3} - \dfrac{2-2\sqrt{2}}{2} =$

$\dfrac{2\left(2\sqrt{3}+3\right) - 3\left(2-2\sqrt{2}\right)}{6}$

$= \dfrac{4\sqrt{3}+6-6+6\sqrt{2}}{6} = \dfrac{4\sqrt{3}+6\sqrt{2}}{6}$

$= \dfrac{4\sqrt{3}}{6} + \dfrac{6\sqrt{2}}{6} = \dfrac{2}{3}\sqrt{3}+\sqrt{2}$

b) Method 1

Let cost price $= x$

Profit is 15 %

Hence the selling price corresponds to

$\dfrac{100\%+15\%}{100\%} \times x = \dfrac{115x}{100}$

The buyer later sold it to Mr. Nana at a loss of 10 % implies that Mr. Nana paid:

$\dfrac{100\%-10\%}{100\%} \times \dfrac{115x}{100} = \dfrac{90}{100} \times \dfrac{115x}{100}$

But Mr. Nana paid GH¢20,700.00 for the machine.

$\Rightarrow \dfrac{90}{100} \times \dfrac{115x}{100} = 20,700$

$\therefore x = 20,700 \times \dfrac{100}{90} \times \dfrac{100}{115} = 20,000.00$

Hence cost price $= x =$ GH¢ 20,000.00

Method 2

Let cost price $= x$ and selling price $= y$

$x = \dfrac{100y}{115}$

$y - 20,700 = \dfrac{10}{100} \times y$

$\Rightarrow 100y - 2,070,000 = 10y$

$90y = 2,070,000$

$y = \dfrac{2,070,000}{90} = 23,000$

$\therefore x = \dfrac{100y}{115} = \dfrac{100 \times 23,000}{115} = 20,000$

Hence cost price $= x =$ GH¢ 20,000.00

Question 2

a)
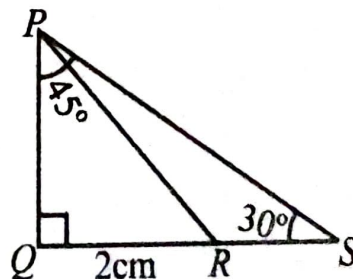
Not drawn to scale

In the diagram, $|QR| = 2$ cm, $\angle PQR = 90°$, $\angle RSP = 30°$ and $\angle QPR = 45°$.

Find:

i) $|PR|$

ii) $|RS|$ in surd form (radicals)

b)
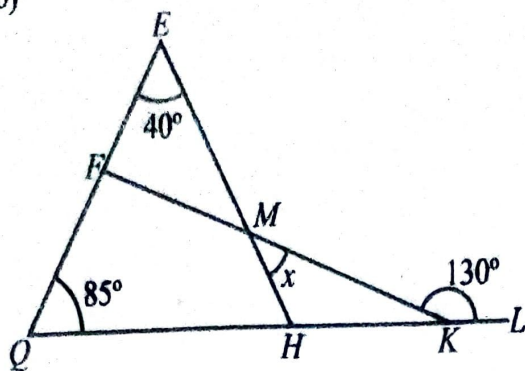
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Find the size of the angle marked x in the diagram.

Solution

a) i) *Finding the length of PR*
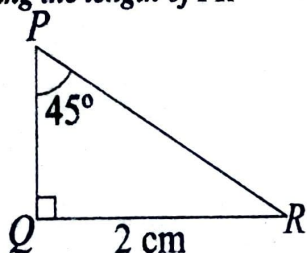
$$\sin 45° = \frac{2}{|PR|} \qquad \Rightarrow |PR| = \frac{2}{\sin 45°} = \frac{2}{0.7071}$$

$$\therefore |PR| = 2.8285 = 2.83 \text{ cm}$$

$$Or \ |PR| = \frac{2}{\sin 45°} = \frac{2}{\frac{\sqrt{2}}{2}} = 2\sqrt{2} \text{ cm}$$

ii) It can be deduced that $\triangle PQR$ is an isosceles triangle and so $|PQ| = 2$ cm. This is illustrated in the following figure
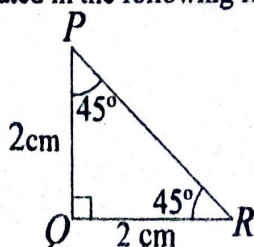
Consider triangle PQS, as follows:
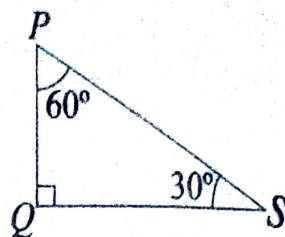
From $\triangle PQS$

$$\tan 60° = \frac{|QS|}{2} \qquad \Rightarrow |QS| = 2 \tan 60°$$

$$\therefore |QS| = 2 \times \sqrt{3} = 2\sqrt{3} \text{ cm}$$

From the given figure,

$$|RS| = |QS| - |QR| = \left(2\sqrt{3} - 2\right) \text{ cm}$$

$$\therefore |RS| = 2\left(\sqrt{3} - 1\right) \text{ cm}$$

b) *Finding angle EHK (Consider triangle EQH)*

Let $\angle EHK = y$
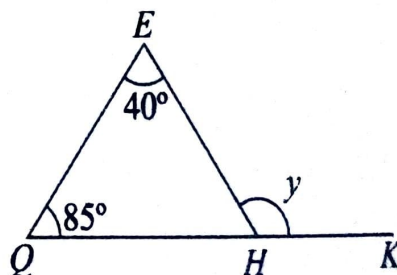
From the exterior angle theorem $\left(\text{using } \triangle EQH\right),$

$$y = 40° + 85° \qquad\qquad \therefore y = 125°$$

b) *Finding x (Consider triangle MHK)*
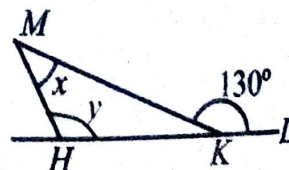
From the exterior angle theorem
(using $\triangle MHK$),

$x + y = 130°$

$x + 125° = 130°$

$\therefore x = 130° - 125° = 5°$

Question 3

The distribution of the ages of 40 students in a class is as follows:

Age (years)	14	15	16	17	18	19	20	21	22
Number of students	1	3	4	6	10	6	5	3	2

Find the:
a) mean age;
b) inter-quartile range.

Solution

x	f	fx
14	1	14
15	3	45
16	4	64
17	6	102
18	10	180
19	6	114
20	5	100
21	3	63
22	2	44
	$\sum f = 40$	$\sum fx = 726$

a) *Finding the mean age:*

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{726}{40} = 18.15 \text{ years}$$

b) *Finding the inter-quartile range*
Step 1 (Find the lower quartile):

$$\frac{\sum fx}{4} = \frac{40}{4} = 10$$

This implies that the lower quartile corresponds to a total frequency of 10. Hence from the table, the lower quartile is 17 years.

Step 2 (Find the upper quartile):

$$\frac{3 \times \sum fx}{4} = \frac{3 \times 40}{4} = 30$$

This implies that the upper quartile corresponds to a total frequency of 30. Hence from the table, the upper quartile is 19 years.

Step 3: Determine the inter-quartile range
Inter-quartile range
= (Upper quartile) – (Lower quartile)
= (19 years) – (17 years) = 2 years

Question 4
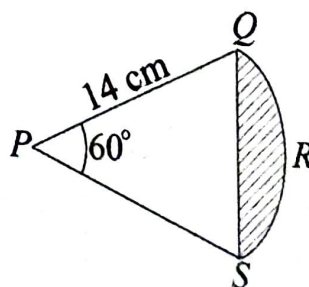
The diagram shows a sector of a circle centre P and radius 14cm. If QS is a chord and $\angle QPS = 60°$, calculate, correct to one decimal place, the area of the shaded region.
[Take $\pi = \frac{22}{7}$]

Solution

Step 1: Finding the area of $\triangle PQS$

Let h = height of $\triangle PQS$
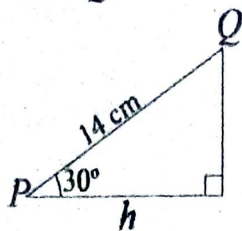
From the figure, $\cos 30° = \dfrac{h}{14}$

$\therefore$ Height of $\triangle PQS$ is $h = 14 \cos 30°$

Height of $\triangle PQS = 14 \times \frac{\sqrt{3}}{2} = 7\sqrt{3}$ cm

OR Height of $\triangle PQS = 12.1244$ cm

Area of $\triangle PQS = \dfrac{1}{2} \times (\text{base}) \times (\text{height})$

$= \dfrac{1}{2} \times 14 \times 12.1244$

$\therefore$ Area of $\triangle PQS = 84.8705$ cm^2

Step 2: Finding the area of sector PQS

Area of sector $PQS = \dfrac{\theta}{360} \times \pi R^2$

Area of sector $PQS = \dfrac{60°}{360°} \times \dfrac{22}{7} \times 14^2$

$\therefore$ Area of sector $PQS = 102.6667$ cm^2

Step 3: Finding the area of the shaded portion

Area of shaded portion

$= (\text{Area of sector } PQR) - (\text{Area of } \triangle PQS)$

$\therefore$ Area of shaded portion

$= (102.6667 - 84.8705)$ cm^2

$= 17.8$ cm^2 (1 d.p)

Question 5

a) Solve $\frac{x}{3} - \frac{1}{4}(x+2) > 3x - 2\frac{1}{5}$.

b) In the diagram $PS \parallel QR$, $\angle PSR = 79°$, $\angle SPR = 54°$ and TQ is a tangent to the circle at P.
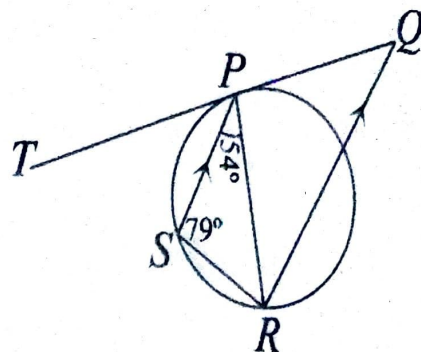
Not drawn to scale

Calculate:

i) $\angle TPS$; ii) $\angle PQR$

Solution

a) $\frac{x}{3} - \frac{1}{4}(x+2) > 3x - 2\frac{1}{5}$

$\frac{x}{3} - \frac{1}{4}(x+2) > 3x - \frac{11}{5}$

Multiply through by 60:

$60 \times \left[\frac{x}{3} - \frac{1}{4}(x+2)\right] > 60 \times \left[3x - \frac{11}{5}\right]$

$20x - 15(x+2) > 180x - 132$

$20x - 15x - 30 > 180x - 132$

$-30 + 132 > 180x - 20x + 15x$

$102 > 175x$

$\dfrac{102}{175} > \dfrac{175x}{175}$

$0.5829 > x$

Or $x < 0.5829$

b) i) $79° + 54° + \angle PRS = 180°$

$133° + \angle PRS = 180°$

$\therefore \angle PRS = 180° - 133° = 47°$

$\angle TPS = \angle PRS = 47°$

($\angle$s in alternate segment are equal)

ii) $\angle TPS = \angle PQR$ (corresponding $\angle$s are equal)

$\therefore \angle PQR = 47°$

Question 6

a) A trader sold an article at a discount of 8% for GH¢828.00. If the article was initially marked to gain 25 %, find the:
   i) cost price of the article;
   ii) discount allowed.

b) X varies directly as the cube of Y and inversely as the square root of Z.
   If $X = 108$ when $Y = 3$ and $Z = 4$, find Z when $X = 4000$ and $Y = 10$.

Solution

a) i) Let x be the cost of the article before the discount.

A discount of 8% was given when selling price is GH¢828.00 (i.e. 100% – 8%= 92%)

$$\Rightarrow \frac{92x}{100} = 828 \qquad \Rightarrow x = \frac{828 \times 100}{92}$$

$$\therefore x = GH \text{ } ¢900.00$$

Let y be the cost of article with initial marked gain of 25 %

$$\Rightarrow \frac{125y}{100} = 900 \qquad \Rightarrow y = \frac{900 \times 100}{125}$$

$$\therefore y = GH \text{ } ¢720.00$$

ii) Discount allowed $= x - y$
   $= GH \text{ } ¢(900 - 828)$
   $\therefore$ Discount allowed $= GH \text{ } ¢72.00$

b) $X \alpha \dfrac{Y^3}{\sqrt{Z}} \Rightarrow X = \dfrac{kY^3}{\sqrt{Z}}$ ------(1)

Substituting $X = 108$, $Y = 3$ and $Z = 4$ into equation (1) gives:

$$108 = \frac{k \times (3)^3}{\sqrt{4}} \qquad \Rightarrow 108 = \frac{k \times 27}{2}$$

$$\therefore k = \frac{108 \times 2}{27} = 8 \Rightarrow X = \frac{8Y^3}{\sqrt{Z}}$$

Substituting $X = 4000$, $Y = 10$ and $k = 8$ into equation (1) gives:

$$4000 = \frac{8 \times (10)^3}{\sqrt{Z}} \qquad \Rightarrow 4000 = \frac{8000}{\sqrt{Z}}$$

$$\Rightarrow \sqrt{Z} = \frac{8000}{4000} = 2$$

$$\therefore Z = 2^2 = 4$$

Question 7

A ship sailed 5120 km from port X to port Y on bearing of 345°. It then sailed 448 km on a bearing of 075° to port Z. Calculate, correct to the nearest whole number, the:
a) distance from Z to X;
b) bearing of Z from X;
c) time it took to sail from X to Z through Y at a constant speed of 120 km/h.

Solution

a) The following figure is an illustration of the given problem:
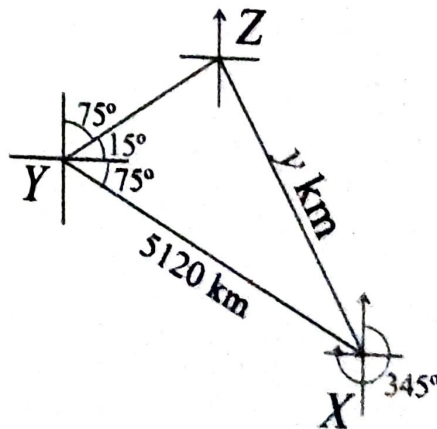
From the diagram,
$\angle XYZ = 15° + 75° = 90°$
Hence ΔXYZ is a right angled triangle as
Hence triangle XYZ is a right-angled triangle as shown in the following diagram:
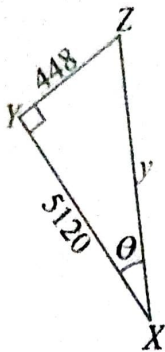
Let y = distance from Z to X and
θ = angle between XY and XZ
From Pythagoras theorem,
$y^2 = 5120^2 + 448^2$
$y^2 = 26,214,400 + 200,704 \Rightarrow y^2 = 26,415,104$
$\therefore y = \sqrt{26,415,104} = 5,139.56$
Hence the distance from Z to X is 5,140 km

b) Finding the bearing of Z from X
From the diagram,
$\tan\theta = \dfrac{448}{5120} \qquad \Rightarrow \tan\theta = 0.0875$
$\therefore \theta = \tan^{-1}(0.0875) \qquad \Rightarrow \theta = 5.00°$
The bearing of Z from X is:
Bearing $= 345° + 5° = 350°$
Now $360° - 10° = 350°$
Hence bearing of Z from X is N 10° W

c) *Finding the time taken (Method 1)*
Total distance from X to Z through Y is:
Distance $= 5120 + 448 = 5568$ km
Speed $= 120$ km/h
$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}} \Rightarrow \text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$
$\therefore \text{Time} = \dfrac{5568 \text{ km}}{120 \text{ km/h}} = 46.4$ hours
Hence the time taken is 46 hours

Finding the time taken (Method 2)
Time taken to sail from X to Y is:
$\text{Time} = \dfrac{5120 \text{ km}}{120 \text{ km/h}} = 42.67$ hours
Time taken to sail from Y to Z is:
$\text{Time} = \dfrac{448 \text{ km}}{120 \text{ km/h}} = 3.73$ hours
Total time taken $= 42.67 + 3.73$
$= 46.4$ hours

Question 8
a) i) Make q the subject of the relation:
$r = \dfrac{f}{2} + \left(\dfrac{f^2}{4} + q^2\right)^{\frac{1}{2}}$
ii) Find, correct to 3 significant figures, the positive value of q when $f = 4$ and $r = 5$.

b) A student plans to spend ₦200.00 on p notebooks. But the price of the notebooks had increased by ₦10.00. As a result, the number of notebooks the student could buy was reduced by 1. Find the price of **each** notebook before the increase.

Solution
Method 1
a) i) $r = \dfrac{f}{2} + \left(\dfrac{f^2}{4} + q^2\right)^{\frac{1}{2}} \Rightarrow r - \dfrac{f}{2} = \sqrt{\dfrac{f^2}{4} + q^2}$
Squaring both sides gives:
$\left(r - \dfrac{f}{2}\right)^2 = \left(\dfrac{f^2}{4} + q^2\right)$
$q^2 = \left(r - \dfrac{f}{2}\right)^2 - \dfrac{f^2}{4}$
$\therefore q = \pm\sqrt{\left(r - \dfrac{f}{2}\right)^2 - \dfrac{f^2}{4}}$

ii) When $f = 4$ and $r = 5$,

$$q = \sqrt{\left(5 - \frac{4}{2}\right)^2 - \frac{4^2}{4}} = \sqrt{3^2 - 4} = \sqrt{5}$$

$$\therefore q = 2.24 \quad (3\text{s.f})$$

Method 2

i) Squaring both sides gives:

$$\left(r - \frac{f}{2}\right)^2 = \left(\frac{f^2}{4} + q^2\right)$$

$$r^2 - 2r\left(\frac{f}{2}\right) + \left(\frac{f}{2}\right)^2 = \frac{f^2}{4} + q^2$$

$$r^2 - fr + \frac{f^2}{4} = \frac{f^2}{4} + q^2$$

$$r^2 - fr = q^2$$

$$\therefore q = \pm\sqrt{r^2 - fr}$$

ii) When $f = 4$ and $r = 5$,

$$q = \sqrt{5^2 - 4 \times 5} = \sqrt{25 - 20} = \sqrt{5} = 2.24 \quad (3\ \text{s.f})$$

b) The price of each notebook before the

$$\text{increase} = \frac{200}{p}$$

Since the number of notebooks the student could buy was reduced by 1, we have:

$$\frac{200}{p-1} - \frac{200}{p} = 10$$

$$p(p-1) \times \left(\frac{200}{p-1} - \frac{200}{p}\right) = p(p-1) \times 10$$

$$200p - 200(p-1) = 10p(p-1)$$

$$200p - 200p + 200 = 10p^2 - 10p$$

$$10p^2 - 10p - 200 = 0$$

$$p^2 - p - 20 = 0$$

$$(p+4)(p-5) = 0$$

Either $p + 4 = 0$ given $p = -4$; which is out.

Or $p - 5 = 0$ given $p = 5$

The price of each notebook before the

$$\text{increase} = \frac{200}{p} = \frac{200}{5} = ₦40.00$$

Question 9

The boundaries of a farmland are marked by trees which are located at points P, Q and R such that $|QR|$ = 850 m, $\angle PQR$ = 60° and $\angle PRQ$ = 45°. A poultry is to be located at a point S within the farmland such that S is 400m from P and equidistant from Q and R.

a) Using ruler and a pair of compasses only and a scale of 1cm to 100 m:
i) construct the boundaries of the farmland.
ii) locate the point S.

b) Measure: i) $|PR|$ ii) $|SR|$

Solution

$\angle PQR = 60°$, $\angle PRQ = 45°$, $|QR| = 8.5$ cm

We need to construct part of a circle which has P as the centre and radius 4 cm as shown in the following figure:
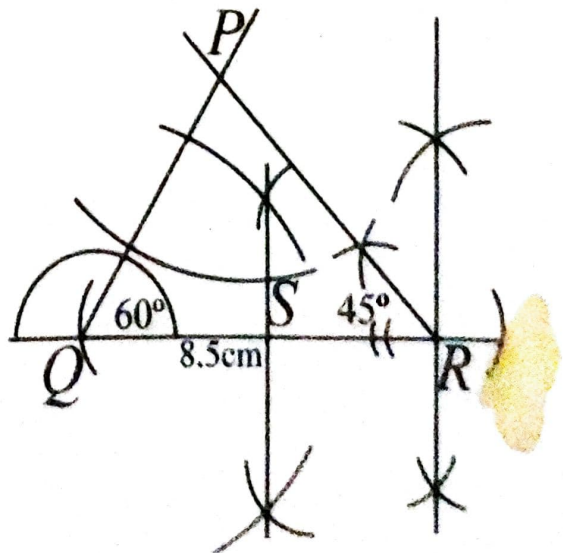
b) i) $|PR|$ = 7.5 cm or 750 m

ii) $|SR|$ = 4.5 cm or 450 m

Question 10

Three bags labelled P, Q and R contains red, blue and white balls respectively of equal sizes. The ratio of the balls in the bags are P : Q = 2 : 3 and Q : R = 4 : 5. All the balls are removed into a big bag and properly mixed together.

a) Find the probability of picking a red ball.

b) If two balls are picked at random one after the other with replacement, find the probability of picking.

  i) a white ball and a blue ball;

  ii) a blue ball **first** and then a red ball.

Solution

$P : Q = 2 : 3 \Rightarrow P = \dfrac{2Q}{3}$

$Q : R = 4 : 5 \Rightarrow R = \dfrac{5Q}{4}$

$\therefore \ P : Q : R = \dfrac{2Q}{3} : Q : \dfrac{5Q}{4} = 8Q : 12Q : 15Q$

Hence total number of balls

$= 8Q + 12Q + 15Q = 35Q = 35Q$

a) The probability of picking a red ball is:

$Pr\,(red) = \dfrac{8}{35} = 0.2286$

b) i) Probability of picking a white ball and a blue ball is:

$Pr(white \ and \ blue)$

$= Pr\left(1^{st} \ is \ white \ and \ 2^{nd} \ is \ blue\right)$

$+ Pr\left(1^{st} \ is \ blue \ and \ 2^{nd} \ is \ white\right)$

$= \dfrac{15}{35} \times \dfrac{12}{35} + \dfrac{15}{35} \times \dfrac{12}{35}$

$= 2\left(\dfrac{15}{35} \times \dfrac{12}{35}\right) = \dfrac{72}{245} = 0.2939$

ii) Probability blue ball first and then a red ball

$= Pr(blue \ and \ red) = \dfrac{12}{35} \times \dfrac{8}{35} = \dfrac{96}{1225} = 0.0784$

Question 11

a)
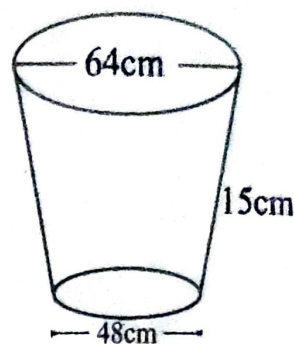
Not drawn to scale

The diagram shows a flower pot in the shape of a frustum of a cone. The diameters of the top and bottom ends are 64 cm and 48 cm respectively. If the slant height is 15cm, calculate the curved surface area of the pot.

[Take $\pi = \frac{22}{7}$ and R = 6400km]

b) From town **M** on latitude 60°S, a ship sails 440 km due West along the parallel of latitude to a town N. Find, correct to the **nearest degree**, the angle subtended by arc MN at the centre of the parallel of latitude.

Solution

a) The following is an illustration of the entire cone used to form the frustum:
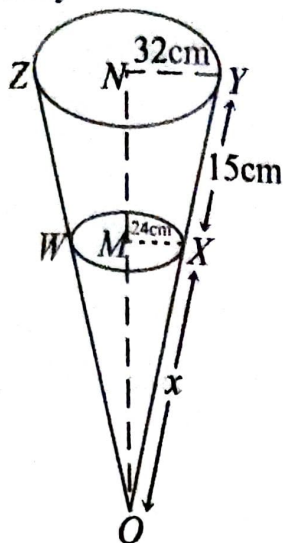
Since $\triangle OMX$ is similar to $\triangle ONY$

$$\Rightarrow \frac{x}{x+15} = \frac{24}{32} \quad \Rightarrow 32x = 24(x+15)$$

$$32x = 24x + 360 \quad \Rightarrow 8x = 360$$

$$\therefore x = \frac{360}{45} = 45\,cm$$

Curved surface area of smaller cone

$$= \pi rx = \frac{22}{7} \times 24 \times 45 = 3394.2857 cm^2$$

Curved surface area for bigger cone

$$= \pi r(15) = \frac{22}{7} \times 32 \times 60 = 6034.2857 cm^2$$

Curved surface area of frustum
= The difference between the curved
   surface area for bigger cone and the
   curved surface area for smaller cone

$$= 6034.2857\,cm^2 - 3394.2857\,cm^2$$

$$= 2640 \ cm^2$$

b) The voyage is illustrated as follows:
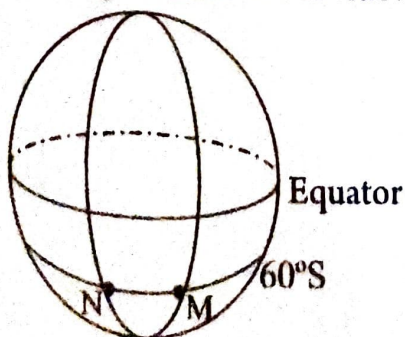
Let R = radius of the earth
θ = the angle subtended by arc MN at the
   centre of the parallel of latitude

Length of arc MN $= \dfrac{\theta}{360} \times 2\pi R \cos\theta$

$$\Rightarrow \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6400\cos 60^\circ = 440$$

$$\therefore \theta = \frac{440 \times 360 \times 7}{2 \times 22 \times 6400\cos 60^\circ} = 7.875 \approx 8^\circ$$

Question 12

a) Using a scale of 2cm to 2 units on both axes,
   draw on a sheet of graph paper two
   perpendicular axes $0x$ and $0y$ for $-10 \le x \le 10$
   and $-12 \le y \le 12$.

b) Given the point $A(2, 3)$ and the vectors
   $\vec{AB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\vec{BC} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$, draw on same
   graph sheet, indicating clearly all vertices
   and their coordinates:
   i) triangle ABC;
   ii) the image $\triangle A_1B_1C_1$ of $\triangle ABC$ under a
       reflection in the line $x - 4 = 0$ where
       $A \to A_1$, $B \to B_1$ and $C \to C_1$.

c) Using the graph, calculate $|\vec{A_1C_1}|$, leaving
   the answer in the form $p\sqrt{q}$ where p and q
   are positive integers.

Solution
a) Refer to the graph.

b) i) Finding the coordinates of the vertices

$$\vec{AB} = \vec{OB} - \vec{OA} \Rightarrow \vec{OB} = \vec{AB} + \vec{OA} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\vec{BC} = \vec{OC} - \vec{OB} \Rightarrow \vec{OC} = \vec{BC} + \vec{OB} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

Hence we have: $A(2, 3)$, $B(3,5)$ and $C(-1, 6)$

ii) Reflection in the line x − 4 = 0

$$\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{\text{Reflection in line } x-4=0} \begin{pmatrix} 8-x \\ y \end{pmatrix}$$

$$A\begin{pmatrix} 2 \\ 3 \end{pmatrix} \rightarrow A_1\begin{pmatrix} 8-2 \\ 5 \end{pmatrix} = A_1\begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$B\begin{pmatrix} 3 \\ 5 \end{pmatrix} \rightarrow B_1\begin{pmatrix} 8-3 \\ 5 \end{pmatrix} = B_1\begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

$$C\begin{pmatrix} -1 \\ 6 \end{pmatrix} \rightarrow C_1\begin{pmatrix} 8--1 \\ 6 \end{pmatrix} = C_1\begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

Hence we have:

$A_1(6, 3)$, $B_1(5, 5)$ and $C_1(9, 6)$

The graph is as follows:
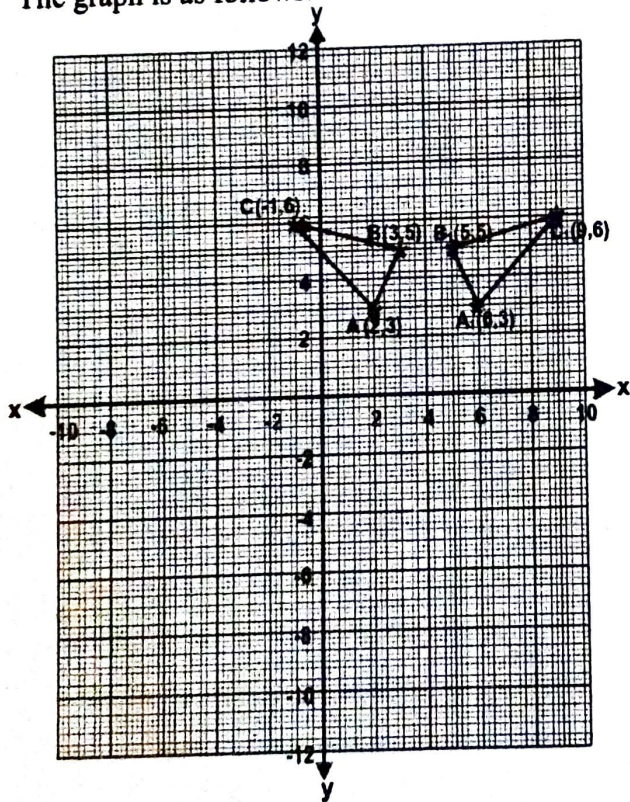
c) $|\overrightarrow{A_1C_1}| = \sqrt{(6-9)^2 + (3-6)^2} = \sqrt{9+9} = \sqrt{18}$

$\therefore |\overrightarrow{A_1C_1}| = 3\sqrt{2}$ units

(i.e $p = 3$ and $q = 2$)

Question 13

a) For what values of x would the matrix $\begin{pmatrix} x-10 & -6 \\ 4 & x+1 \end{pmatrix}$ have **no** inverse?

b) i) Using a scale of 2 cm to 1 unit on both axes, draw the graph of $y - 3 = 0$, $y = 6 - x$ and $x = 5$.

ii) shade the region which satisfies the inequalities.

$y - 3 \leq 0$

$y \geq 6 - x$ and

$x \leq 5$.

Solution

a) Let $A = \begin{pmatrix} x-10 & -6 \\ 4 & x+1 \end{pmatrix}$

The determinant of the given matrix is:

$|A| = (x-10)(x+1) - 4(-6)$

$|A| = x^2 + x - 10x - 10 + 24 \Rightarrow |A| = x^2 - 9x + 14$

For $\begin{pmatrix} x-10 & -6 \\ 4 & x+1 \end{pmatrix}$ to have **no** inverse, $|A| = 0$

$\Rightarrow x^2 - 9x + 14 = 0 \Rightarrow (x-2)(x-7) = 0$

$\therefore x = 2$ or $x = 7$

b) i) The graph of $y - 3 = 0$ (i.e. $y = 3$) is the horizontal line through ($y = 3$). Refer to graph).

The graph of $x = 5$ is the vertical line through $x = 5$. Refer to (the graph).
For the graph of $y = 6 - x$; when $x = 1$, $y = 5$ and when $x = 0$, $y = 6$.

The required graph is as follows:
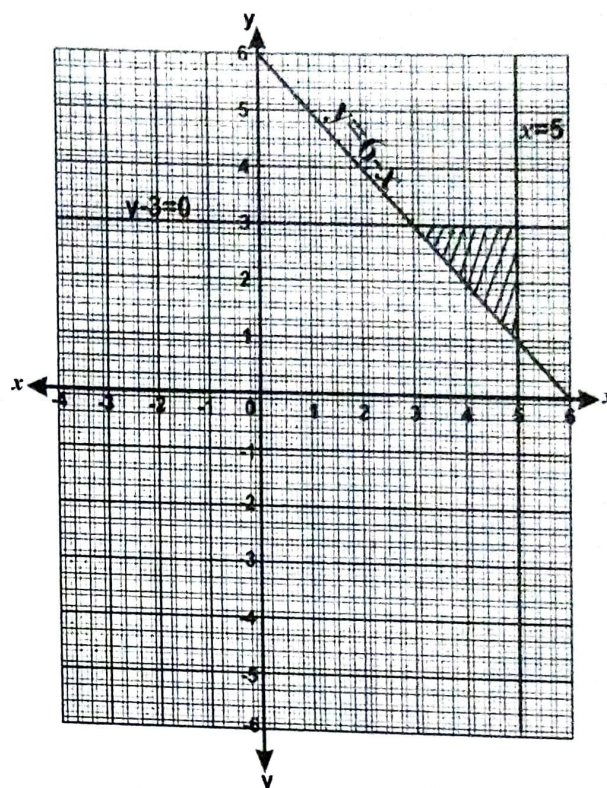
ii) The region which satisfies the inequalities is shaded on the graph.

41 JUNE 2018 OBJECTIVE TEST

1. Simplify: $\dfrac{\frac{1}{4} \times 2\frac{1}{2}}{12 \div 1\frac{1}{2}}$

   A. $\frac{5}{8}$

   B. $\frac{5}{32}$

   C. $\frac{5}{64}$

   D. $\frac{5}{72}$

2. Given that $3 \times 9^{1+x} = 27^{-x}$, find the value of x.

   A. $-\frac{3}{5}$

   B. $-1\frac{2}{3}$

   C. -2

   D. -3

3. In a class of 39 students, 25 offer Fante and 19 offer Twi. Five students do not offer any of the two languages. How many students offer only Twi?

   A. 6

   B. 7

   C. 8

   D. 9

4. Evaluate $\dfrac{4 - \sqrt{2}}{\sqrt{2}}$.

   A. $\sqrt{2} - 1$

   B. $2\sqrt{2} - 1$

   C. $\sqrt{2} + 1$

   D. $2\sqrt{2} + 1$

5. Mary and Charity entered into a business partnership and agreed to share their profit in the ratio 4:5 respectively. If Mary received GH¢ 5,000.00 less than Charity, how much profit did they make?

   A. GH¢ 30,000.00
   B. GH¢ 35,000.00
   C. GH¢ 40,000.00
   D. GH¢ 45,000.00

6. A ____ ought a car which costs N5,____.00 from a dealer on hire purchase. He pays a deposit of N3,000,000.00 and agrees to pay the balance at 8 % compound interest per

annum. If he pays N1,000,000.00 at the end of each year, how much will be remaining to be paid after two years?

   A. N92,800.00

   B. N252,800.00

   C. N320,000.00

   D. N332,800.00

7. y varies inversely as the square of x. When $x = 3$, $y = 100$. Find the value of x when $y = 25$.

   A. $x = 30$

   B. $x = 12$

   C. $x = 6$

   D. $x = 5$

8. Solve the equation: $t - \frac{9}{5} = -1\frac{1}{15}$

   A. $t = \frac{3}{5}$

   B. $t = \frac{11}{15}$

   C. $t = \frac{4}{5}$

   D. $t = \frac{13}{15}$

9. Find the truth set of the equation:
   $(x-2)^2 + 3 = (x+1)^2 - 6$

   A. $\{-2\}$

   B. $\{-1\}$

   C. 1

   D. $\{2\}$

10. Two friends, Dede and Kofi decided to buy the same type of car. They found out that the car cost N3,000,000.00. The amount of money (f) which Dede had was not enough to buy the car but Kofi had enough money (k) to buy the car. Which of the following inequalities is true?

    A. $f \le N3,000,000.00 \le k$

    B. $f > N3,000,000.00 > k$

    C. $f < N3,000,000.00 \le k$

    D. $f \ge N3,000,000.00 > k$

11. Simplify: $\dfrac{z}{z+2} - \dfrac{z-2}{z-3}$

    A. $\dfrac{4 - 3z}{(z+2)(z-3)}$

    B. $\dfrac{3z - 4}{(z-3)(z+2)}$

    C. $\dfrac{4 + 3z}{(z+3)(z-2)}$

    D. $\dfrac{4 + 3z}{(z+2)(z-3)}$

12. Simplify: $\dfrac{x^{-1} + y^{-1}}{x + y}$

A. $\dfrac{y}{x}$ B. $\dfrac{x}{y}$ C. $\dfrac{1}{xy}$ D. xy

13. A chord of a circle with radius 5 cm subtends an angle of 70° at the centre. Find, correct to one decimal place, the length of the chord.

A. 8.2 cm   B. 5.7 cm
C. 4.1 cm   D. 2.9 cm

14. A pyramid with a square base has a volume of 1,400 cm³. If a side of the base is 10 cm long, find the height of the pyramid.

A. 70 cm   B. 42 cm
C. 21 cm   D. 7 cm

15.
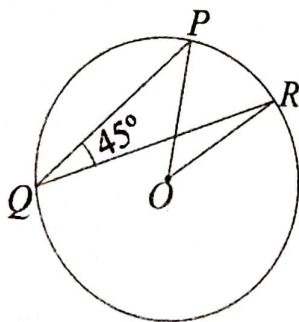
In the diagram, O is the centre of the circle. If the radius of the circle is 12 cm and $\angle PQR = 45°$, calculate, correct to the nearest cm, the length of arc PR.

[Take $\pi = \frac{22}{7}$]

A. 113 cm   B. 57 cm
C. 19 cm   D. 15 cm

16. Which of the following about parallelograms is **true**?

A. Opposite angles are supplementary
B. Opposite angles are complementary
C. Opposite angles are equal
D. Opposite angles are reflex angles

17.
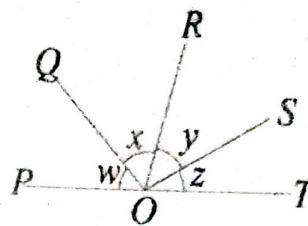
In the diagram, POT is a straight line. If $(w + x + y) = 140°$ and $(x + y + z) = 130°$, find the value of $(x + y)$.

A. 40°   B. 50°
C. 90°   D. 110°

18. The interior angles of a pentagon are $x°$, $(x + 5)°$, $(x + 10)°$, $(x + 15)°$ and $(x + 20)°$. Find the value of x.

19.
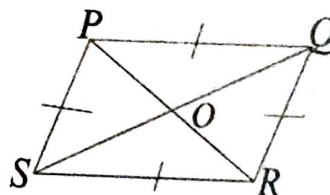
Not drawn to scale

The diagram shows a rhombus $PQRS$ with diagonals intersecting at O.
If the ratio of $\angle PQR$ to $\angle QPS$ is 3 : 7, calculate the value of $\angle QSR$.

A. 27°   B. 45°
C. 54°   D. 63°

20. Calculate the gradient of the line which passes through the points (1, 4) and (−2, 6).

A. $-\frac{3}{2}$ B. $-\frac{2}{3}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$

21. Calculate the distance between points (3, −2) and (8, 10).

A. 12 units   B. 13 units
C. 14 units   D. 15 units

22. Evaluate $\dfrac{\cos 65°}{\sin 25°} + \dfrac{\sin 35°}{\cos 55°}$

   A. 3   B. 2   C. 1   D. 0

23. From a height of 2 m above the ground and at a horizontal distance of $12\sqrt{3}$ m from a tree, the angle of elevation of the top of the tree is $30°$. How tall is the tree?

   A. 8 m   B. $8\sqrt{3}$ m   C. 14 m   D. 18 m

24. Express the bearing of $312°$ in compass direction form.

   A. S 48° W   B. N 48° W
   C. S 48° E   D. N 48° E

25. A fair die is thrown once. What is the probability of obtaining 3?

   A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. 1

The table shows the distribution of the ages of members of a school choir. *Use the table to answer questions* **26** *and* **27**.

Age(years)	15	16	17	18
No. of students	5	6	3	1

26. Calculate the mean age.
   A. 15 years   B. 16 years
   C. 17 years   D. 18 years

27. Find the median age.
   A. $15\frac{1}{2}$ years   B. 16 years
   C. $16\frac{1}{2}$ years   D. 17 years

28. If $x = \dfrac{mn}{3}$ and $m = \dfrac{v}{y}$, express x in terms of v, y and n.

   A. $x = \dfrac{3vy}{n}$ B. $x = \dfrac{vy}{3n}$

   C. $x = \dfrac{vyn}{3}$ D. $x = \dfrac{vn}{3y}$

29. If $M_{six} = 123_{five}$, find the value of M.
   A. 53   B. 55
   C. 62   D. 102

30. In triangle XYZ, $|XY| = 8$ cm and Z is equidistant from X and Y. If Z is 5 cm from X, find the area of the triangle.
   A. 24 cm²   B. 18 cm²
   C. 12 cm²   D. 10 cm²

31. Calculate the total surface area of a cuboid whose dimensions are 12 cm × 8cm × 3 cm.
   A. 96 cm²   B. 156 cm²
   C. 288 cm²   D. 312 cm²

32. Calculate, correct to the **nearest** whole number, the volume of a sphere with diameter 6 cm.   [Take $\pi = \frac{22}{7}$]
   A. 15 cm³   B. 36 cm³
   C. 113 cm³   D. 124 cm³

The diagram shows a circle centre O. *Use it to answer questions* **33** *and* **34**.
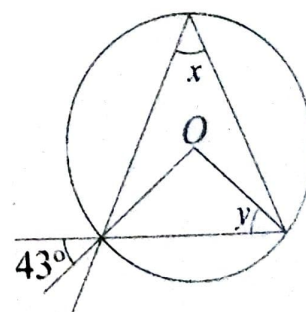
33. Find the value of y.
   A. 43°   B. 47°
   C. 54°   D. 89°

34. Find the value x.
   A. 43°   B. 47°
   C. 54°   D. 89°

35. Find the nth term of the Geometric Progression (G.P.): $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$

A. 2^{n-1}

B. 2^n

C. $\frac{1}{2^n}$

D. $\frac{1}{2^{n-1}}$

36. An old man goes for a walk every 6 days. If he went for a walk on Wednesday, when will he go for the next walk?

A. Monday

B. Tuesday

C. Wednesday

D. Thursday

The cumulative frequency curve (Ogive) shows the distribution of waiting time of some customers at a filling station. *Use the curve to answer questions 37 and 38.*
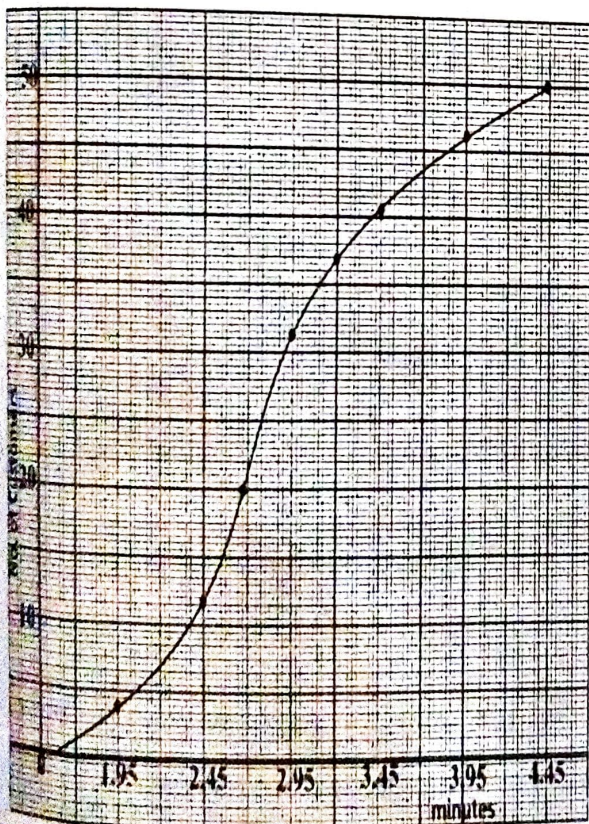
37. Estimate the inter-quartile range.

A. 0.70

B. 0.75

C. 0.80

D. 0.85

38. How many customers waited for **at least** 3 minutes?

A. 16

B. 7

C. 18

D. 19

39. What is the coefficient of x in the expansion of $(4x^2 + 3x - 1)(3x + 1)$?

A. -1

B. 4

C. 1

D. 2

40. Find the quadratic equation whose roots are $\frac{1}{2}$ and $-\frac{3}{2}$.

A. $x^2 + 4x - 3 = 0$

B. $4x^2 + x - 3 = 0$

C. $x^2 - 4x - 3 = 0$

D. $4x^2 + 4x - 3 = 0$

41. Two times a number added to one-third of the number gives $5\frac{1}{6}$. Find the number.

A. $2\frac{2}{7}$

B. $2\frac{3}{14}$

C. $2\frac{1}{7}$

D. $2\frac{1}{14}$

42. Find the mean deviation of 5, 8, 3, 7, 2.

A. 0

B. 2

C. 5

D. 10

43. In a certain community, 1 out of every 8 persons is a graduate. If Yomi and Etteh are members of the community, what is the probability that they are both graduates?

A. $\frac{1}{64}$

B. $\frac{1}{16}$

C. $\frac{1}{8}$

D. $\frac{1}{4}$

44. Which of the following statements is **false**?
 A. In a circle, equal chords subtend equal angles at the centre.
 B. The length of an arc is proportional to the angle subtended by the arc at the centre of the circle.
 C. The circumference of a circle is directly proportional to its diameter.
 D. The angle between the tangent to a circle and its radius is complementary.

45.
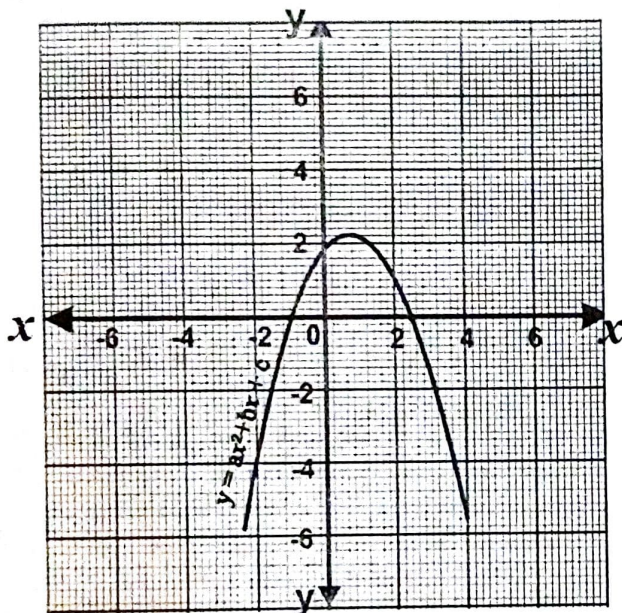
Find the equation of the graph in the diagram.

A. $y = 2 - x - x^2$

B. $y = 2 - x + x^2$

C. $y = 2 + x - x^2$

D. $y = 2 + x + x^2$

46. If $\log x = 0.3030$, $\log y = 0.4771$ and

$\log Z = 0.8451$, evaluate $\dfrac{\log x - \log Z^{\frac{1}{3}}}{\log y^3}$

A. 1.4313   B. 0.8466
C. 0.5466   D. 0.0149

47. Consider these two statements:
 P: *n* is an odd number
 Q: *n* is a prime number greater than 2.

Express "If *n* is not an odd number, then *n* is a prime number greater than 2" in symbolic form.

A. $\sim P \land \sim Q$ B. $\sim P \Rightarrow Q$

C. $P \Rightarrow \sim Q$ D. $\sim P \land \sim Q$

48. Which of the following is **not** a rational number?

A. -5 B. $\sqrt{6}$

C. $3\frac{3}{4}$ D. $\sqrt{90}$

Use the diagram to answer questions 49 and 50.

49. Find the value of *m*.
 A. 35°   B. 45°
 C. 65°   D. 75°

50. Find the value of *n*.
 A. 70°   B. 65°
 C. 55°   D. 35°

2018 THEORY QUESTIONS AND ANSWERS

Question 1

a) Solve correct to **one** decimal place, $\tan(\theta + 25°) = 5.145$, where $0° \leq \theta \leq 90°$.

b) In the relation $t = m\sqrt{n^2 - 4r}$:

   i) make n the subject of the relation.
   ii) find the positive value of n when $t = 25$, $m = 5$ and $r = 4$.

Solution

a) $\tan(\theta + 25°) = 5.145$

$$\theta + 25° = \tan^{-1}(5.145)$$
$$\theta + 25° = 79.00°$$
$$\therefore \theta = 79.00° - 25° = 54.0° \text{ (1 d.p)}$$
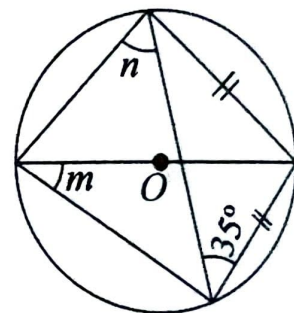
b) i) $t = m\sqrt{n^2 + 4r}$

Squaring both sides gives:

$$t^2 = m^2(n^2 + 4r)$$
$$t^2 = m^2 n^2 + m^2(4r)$$
$$m^2 n^2 = t^2 - 4m^2 r$$
$$n^2 = \frac{t^2 - 4m^2 r}{m^2}$$
$$n = \sqrt{\frac{t^2 - 4m^2 r}{m^2}}$$

ii) When $t = 25$, $m = 5$ and $r = 4$,

$$n = \sqrt{\frac{25^2 - 4(5)^2(4)}{5^2}}$$

$$n = \sqrt{\frac{625 - 25(16)}{25}} = \sqrt{\frac{625 - 400}{25}} = \sqrt{9} = 3$$

Question 2

The first three terms of an Arithmetic Progression (A.P) are $(x+1)$, $(4x-2)$ and $(6x-3)$ respectively. If the last term is 18, find the:

   a) value of x.
   b) sum of the terms of the progression.

Solution

a) Let $U_n = n^{th}$ term

The common difference is given by:
$$U_2 - U_1 = U_3 - U_2$$
$$(4x-2) - (x+1) = (6x-3) - (4x-2)$$
$$4x - 2 - x - 1 = 6x - 3 - 4x + 2$$
$$3x - 3 = 2x - 1$$
$$\therefore x = 2$$

b) Step 1: Finding the number of terms

First term is: $a = U_1 = x + 1 = 2 + 1 = 3$

Second term is $U_2 = 4x - 2 = 4(2) - 2 = 6$

Third term is $U_3 = 6x - 3 = 6(2) - 3 = 9$

The common difference is given by:
$$d = U_2 - U_1 = 6 - 3 = 3$$

The last term is 18 $\Rightarrow U_n = 18$

But $U_n = a + (n-1)d \Rightarrow 18 = 3 + (n-1)(3)$
$$18 = 3 + 3n - 3 \qquad \therefore n = 6$$

Step 2: Finding the sum

The sum of the terms of the progression is given by:

$$S_n = \frac{n}{2}(a + l), \text{ where l is the last term}$$

$$\therefore S_6 = \frac{6}{2}(3 + 18) = 3 \times 21 = 63$$

Question 3

The angle of a sector of a circle with radius 22cm is $60°$. If the sector is folded such that the straight edges coincide, forming a cone, calculate, correct to one decimal place, the:
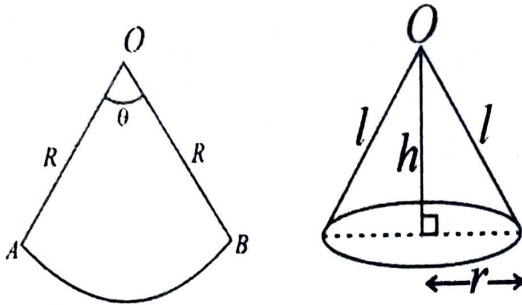
   a) radius;
   b) height;
   c) volume; of the cone. [Take $\pi = \frac{22}{7}$]

Solution

a) Let R = radius of the sector = 22 cm

  θ = angle of the sector = $60°$

  r = radius of the cone = 22 cm

  h = height of the cone

  l = slant height of the cone = 22 cm

Length of arc of = Circumference of base

  sector   of cone

$$\Rightarrow \frac{\theta}{360} \times 2\pi R = 2\pi r$$

$$\therefore r = \frac{\theta}{360} \times R = \frac{60°}{360°} \times 22 = 3.6667 \text{ cm}$$

Hence radius of the cone is 3.7 cm (1 d.p)

b) Applying Pythagoras theorem to the figure,

$$r^2 + h^2 = l^2 \qquad \Rightarrow h^2 = l^2 - r^2$$

$$\Rightarrow h^2 = (22)^2 - (3.6667)^2 \Rightarrow h^2 = 470.5553$$

$$\therefore h = \sqrt{470.5553} = 21.6923 \text{ cm}$$

Hence height of the cone is 21.7 cm (1 d.p)

c) Volume of the cone is given by:

$$V = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3} \times \tfrac{22}{7} \times (3.6667)^2 \times 21.6923$$

$$\therefore V = 305.5341 \approx 305.5 \text{ cm}^3$$

Question 4

a) In how many years will GH¢312.50 invested at 4 % per annum simple interest amount to GH¢500.00?

b)
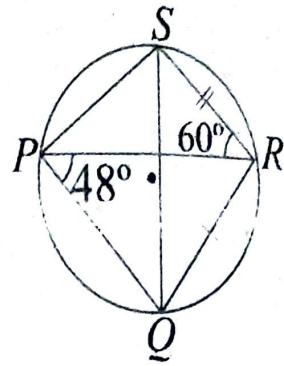
In the diagram, $PQRS$ is a cyclic quadrilateral. If $|SR| = |RQ|$, $\angle SRP = 65°$ and $\angle RPQ = 48°$, find $\angle PRQ$.

Solution

a) Principal is P = GH¢312.50

  Rate is R = 4 %

  Interest is I = (Amount) – (Principal)

  I = GH¢ 500 – GH¢ 312.50

  $\therefore I$ = GH¢ 187.50

$$I = PTR \Rightarrow 187.50 = 312.50 \times T \times \frac{4}{100}$$

where T = number of years;

$$\therefore T = \frac{187.50 \times 100}{312.50 \times 4} = 15 \text{ years}$$

b) Since angles in the same segment are equal, it implies that $\angle RSQ = \angle RPQ = 48°$

Since the base angles of isosceles $\triangle QRS$ are equal, it implies that $\angle SQR = \angle RSQ = 48°$, as shown in the following figure.
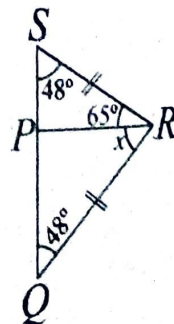
Sum of the angles in $\triangle QRS = 180°$

$\Rightarrow 48° + 48° + 65° + x = 180°$

$\Rightarrow 161° + x = 180°$ $\Rightarrow x = 19°$

$\therefore \angle PRQ = 19°$

Question 5

a) The probabilities that James and Juliet will pass an examination are $\frac{3}{4}$ and $\frac{3}{5}$ respectively. Find the probability that **both** will fail the examination.

b)

Balls	Green	Blue
New	8	2
Old	4	6

The table shows the distribution of balls in a bag. If **2** balls are selected at random **with replacement**, find the probability of selecting **either** 2 new green balls **or** 2 old blue balls.

Solution

a) P(James will pass) $= \frac{3}{4}$

$\Rightarrow$ P(James will fail) $= \frac{1}{4}$

P(Juliet will pass) $= \frac{3}{5} \Rightarrow$ P(Juliet will fail) $= \frac{2}{5}$

P(both will fail)

$=$ P(James will fail **and** Juliet will fail)

$= \frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$

b) Total number of balls $= 8 + 2 + 4 + 6 = $ **20**

There are **8** new green balls.

$\Rightarrow$ Probability (new green ball) $= \dfrac{8}{20} = \dfrac{2}{5}$

There are **6** old blue balls;

$\Rightarrow$ Probability (old blue ball) $= \dfrac{6}{20} = \dfrac{3}{10}$

$\therefore$ Probability (2 new green **or** 2 old blue balls)

$= (\frac{2}{5} \times \frac{2}{5}) + (\frac{3}{10} \times \frac{3}{10})$

$= \frac{4}{25} + \frac{9}{100} = \frac{16+9}{100} = \frac{25}{100} = 0.25$

Question 6

a) If $9^x \times 3^{2y} = \dfrac{1}{729}$, and $2^{-x} \times 4^{-y} = \dfrac{1}{8}$, find the values of x and y.

b) Two commodities X and Y cost D70.00 and D80.00 per kg respectively. If 34.5 kg of X is mixed with 26 kg of Y and the mixture is sold at D85.00 per kg, calculate the percentage profit.

Solution

a) We have been given: $9^x \times 3^{2y} = \dfrac{1}{729}$

$\Rightarrow 3^{2x} \times 3^{2y} = \dfrac{1}{3^6}$ $\Rightarrow 3^{2x+2y} = 3^{-6}$

$\Rightarrow 2x + 2y = -6$ $\Rightarrow x + y = -3 ----- (1)$

We have also been given: $2^{-x} \times 4^{-y} = \dfrac{1}{8}$

$\Rightarrow 2^{-x} \times 2^{2(-y)} = \dfrac{1}{2^3}$ $\Rightarrow 2^{-x-2y} = 2^{-3}$

$\Rightarrow -x - 2y = -3$ $\Rightarrow x = 3 - 2y ----- (2)$

Substituting $x = 3 - 2y$ into equation (1):

$2(3-2y) + 2y = -6$ $\Rightarrow 6 - 4y + 2y = -6$

$\Rightarrow 2y = 12$ $\therefore y = 6$

Substituting $y = 6$ into equation (2) gives:

$x = 3 - 2(6)3 - 12 = -9$

b) Total cost of $X = 70.00 \times 34.5 = $ D 2415.00

Total cost of $Y = 80.00 \times 26 = $ D 2080.00

Total cost of mixture $= 2415.00 + 2080.00$

$\therefore$ Total cost of mixture $= $ D 4495.00

Total sales of mixture $= 85 \times (34.5 + 26)$

Total sales of mixture $= 85 \times 60.5 = $ D 5142.50

Profit $= $ (Sales) $-$ (Cost) $= 5142.50 - 4495.00$

Hence the profit $= $ D 647.50

$\therefore$ Percentage profit $= \dfrac{\text{Profit}}{\text{Cost}} \times 100\%$

$= \dfrac{\text{D } 647.50}{\text{D } 4495.00} \times 100\% = 14.4\%$

Question 7

a) Copy and complete the following table for the relation: $y = 2(x + 2)^2 - 3$ for $-5 \leq x \leq 2$.

X	-5	-4	-3	-2	-1	0	1	2
Y			-1	-3		5		

b) Using scales of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw the graph of the relation $y = 2(x + 2)^2 - 3$ for $-5 \leq x \leq 2$.

c) Use the graph to find the solution of:
   i) $2(x + 2)^2 = 3$;
   ii) $2(x + 2)^2 = 5$.

d) For what values of x, from the graph is y increasing in the interval?

Solution

a) When $x = -5$, $y = 2[(-5) + 2]^2 - 3 = 15$

   When $x = -4$, $y = 2[(-4) + 2]^2 - 3 = 5$

   When $x = -1$, $y = 2[(-1) + 2]^2 - 3 = -1$

   When $x = 1$, $y = 2[(1) + 2]^2 - 3 = 15$

   When $x = 2$, $y = 2[(2) + 2]^2 - 3 = 29$

The complete table is as follows.

x	-5	-4	-3	-2	-1	0	1	2
y	15	5	-1	-3	-1	5	15	29

b) The following is the graph of the relation:
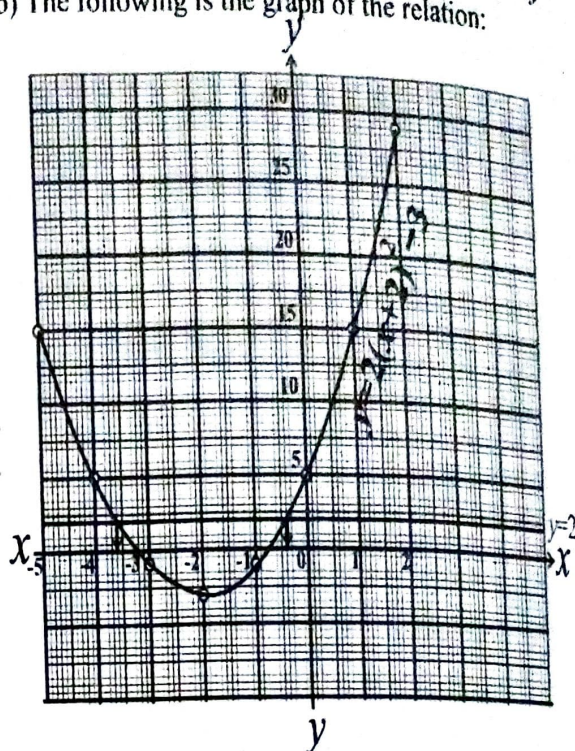
c) i) $2(x + 2)^2 = 3$ $\Rightarrow 2(x + 2)^2 - 3 = 0$

   $\therefore y = 0$ (i.e. the x-axis)

   This implies that the solution of the given equation occurs where the curve drawn cuts the x-axis.

   From the graph, $\{x : x = -3.2, -0.8\}$

ii) $2(x + 2)^2 = 5$

   $\Rightarrow 2(x + 2)^2 - 3 = 5 - 3$ $\Rightarrow y = 2$

   This implies that the solution of the given equation occurs at the points of intersection of the curve $y = 2(x + 2)^2 - 3$) and the horizontal line $y = 2$.

   From the graph, $\{x : x = -3.6, -0.4\}$

d) From the graph, the values of x for which y is increasing in the given interval is:
   $-2 < x \leq 2$

a)

In the diagram, MN // ST, $\angle MNR = 230°$ and $\angle TSR = 76°$. Find the value of $\angle NRS$.

b) Copy and complete the tables for the addition $\oplus$ and multiplication $\otimes$ in modulo 5.

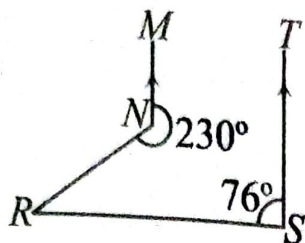| $\oplus$	1	2	3	4
1	2	3	4	0
2	3			
3	4			
4	0			2

$\otimes$	1	2	3	4
1	1	2	3	4
2	2			
3				2
4				1

ii) Use the tables to find:
  α) $4 \otimes 2 \oplus 3 \otimes 4$;
  β) m such that $m \otimes m = m \oplus m$;
  γ) n such that $3 \oplus n = 2 \otimes n$.

Solution

a) *Method 1*

We can produce $\overline{MN}$ to meet $\overline{RS}$ at Q as shown in the following diagram:
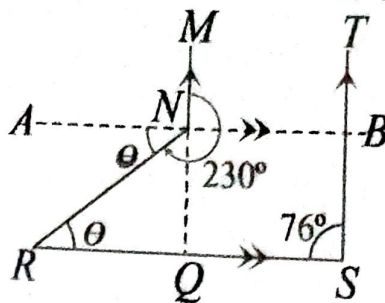
Since line AB is parallel to line RS,
$\Rightarrow \angle MNA = \angle QST = 76°$
From the figure;
$\quad \angle MNA + \angle ANR + 230° = 360°$
$\Rightarrow 76° + \angle ANR + 230° = 360°$
$\Rightarrow \angle ANR = 360° - 76° - 230° = 54°$
But $\angle NRS = \angle ANR = \theta$
$\angle NRS = 54°$

Method 2
$\angle RNQ + 180° = 230°$
$\Rightarrow \angle RNQ = 230° - 180° = 50°$
This is because the total angle on line $MQ = 180°$
Since line AB is parallel to line RS,
$\Rightarrow \angle NQR = \angle TSR = 76°$
Consider $\triangle NQR$ as follows

From the triangle,
$50° + 76° + \angle NRS = 180°$
$126° + \angle NRS = 180°$
$\therefore \angle NRS = 180° - 126° = 54°$

b) i) $2 \oplus 2 = 4$; $2 \oplus 3 = 0$; $2 \oplus 4 = 1$; etc.
$2 \otimes 2 = 4$; $2 \otimes 3 = 1$; $2 \otimes 4 = 3$; etc.

The following are the complete tables:

$\oplus$	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

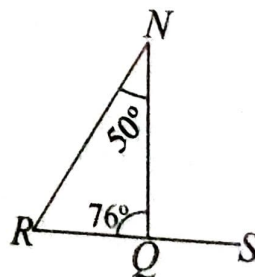| $\otimes$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

ii) α) $4 \otimes 2 \oplus 3 \otimes 4 = 3 \oplus 2 = 0$

β) $m \otimes m = m \oplus m$
But by inspection, $2 \otimes 2 = 2 \oplus 2$
∴ $m = 2$

χ) $3 \oplus n = 2 \otimes n$
But by inspection, $3 \oplus 3 = 2 \otimes 3$ ∴ $n = 3$

Question 9

a) If $16^n = \sqrt[3]{2^2}$, find the value of n.

b) The perimeter of a square and a rectangle is the same. The width of the rectangle is 6 cm and its area is 16 cm² less than the area of the square. Find the area of the square.
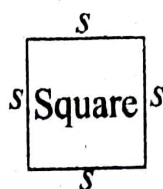
Solution

a) $16^n = \sqrt[3]{2^2}$ $\Rightarrow 2^{4n} = (2)^{\frac{2}{3}}$ $\Rightarrow 4n = \frac{2}{3}$

∴ $n = \frac{2}{3 \times 4} = \frac{1}{6}$

b) Let s = sides of the square;
l = length of the rectangle;
w = width of the rectangle;
as shown in the following figure.

(Perimeter of square) = (Perimeter of rectangle)
$\Rightarrow 4s = 2(l + w)$ $\Rightarrow 4s = 2(l + 6)$
$\Rightarrow 4s = 2l + 12$ ∴ $l = 2s - 6 - - - - - (1)$
(Area of rectangle) = (Area of square) $- 16$
$\Rightarrow lw = s^2 - 16 - - - - - - - - - - - - - (2)$
Substitute $w = 6$ and $l = 2s - 6$ into equation (2):
$\Rightarrow (2s - 6)(6) = s^2 - 16$ $\Rightarrow 12s - 36 = s^2 - 16$
$\Rightarrow s^2 - 12s - 16 + 36 = 0$ $\Rightarrow s^2 - 12s + 20 = 0$

Factorizing gives:
$(s - 2)(s - 10) = 0$
Either $s - 2 = 0 \Rightarrow s = 2$ or
$s - 10 = 0 \Rightarrow s = 10$
But s cannot be 2 since $w = 6$ and s should be greater than w. Hence $s = 10$ cm.
∴ Area of square is $A = s^2 = 10^2 = 100$ cm²

Question 10

The table shows the distribution of marks scored by 500 candidates in an examination.

Marks	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	10	28	40	92	y+60	90	50	30	15	5

a) Find the value of y.
b) Construct a cumulative frequency table and use it to draw a cumulative frequency curve.
c) Use the curve to estimate the probability of selecting a candidate who scored **not more than** 45%.

Solution

a) $\sum f = 500$

$\Rightarrow 10 + 28 + 40 + 92 + y + 60 + 90 + 50 + 30 + 15 + 5 = 500$

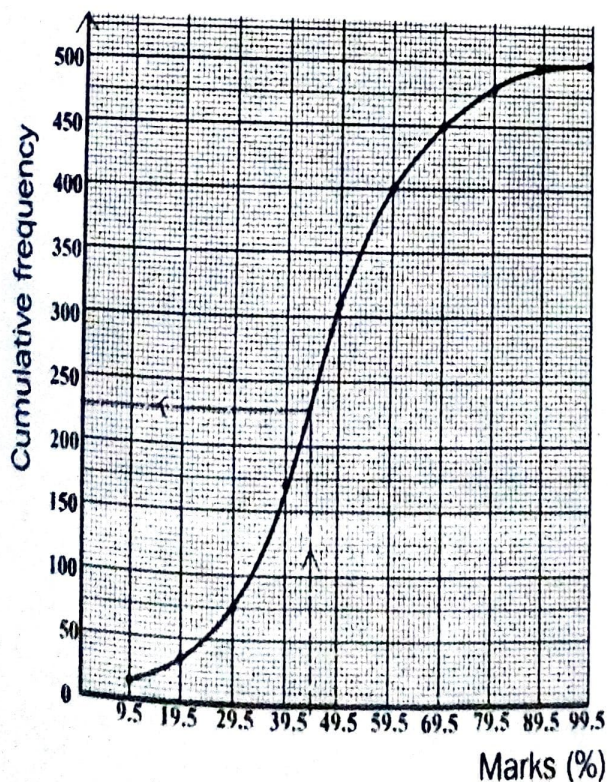$\Rightarrow 420 + y = 500 \qquad\qquad \therefore y = 80$

b) The following is the cumulative frequency table:

Upper Class Boundary	Frequency	Cumulative Frequency
9.5	10	10
19.5	28	38
29.5	40	78
39.5	92	170
49.5	140	310
59.5	90	400
69.5	50	450
79.5	30	480
89.5	15	495
99.5	5	500

The cumulative frequency curve is as follows:

c) Probability $= \dfrac{230}{500} = 0.46$

Question 11

The angle of elevation of the top, X, of a vertical pole from a point, W, on the same horizontal ground as the foot, Z, of the pole is $60°$. If W is 15 km from X and 12 km from a point Y on the pole,
a) illustrate this information with a diagram.
b) calculate, correct to **two** decimal places, the:
  i) angle of elevation of Y from W;
  ii) length, XY.

Solution

a) An illustration of the information is as follows:
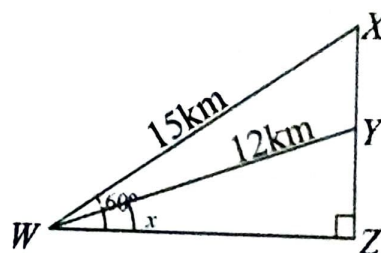
b) i) Calculating the angle of elevation of Y
Using the figure,

$\cos 60° = \dfrac{|WZ|}{15} \quad \Rightarrow |WZ| = 15 \times \cos 60° = 7.5 \ \text{km}$

$\cos x = \dfrac{|WZ|}{12} = \dfrac{7.5}{12} = 0.625$

$\therefore x = \cos^{-1}(0.625) = 51.32°$

Hence the angle of elevation of Y from W is $051.32°$

ii) Calculating the length, XY

$\sin 60° = \dfrac{|XZ|}{15} \quad \Rightarrow |XZ| = 15 \times \sin 60° = 15 \times 0.866$

$\therefore |XZ| = 12.99 \ \text{km}$

$|YZ| = 12 \times \sin 51.32° = 9.3678 \ \text{km}$

$\therefore |XY| = 12.99 - 9.3678 = 3.6222 \approx 3.62 \ \text{km}$

Question 12

a) Using scales of 2 cm to 2 units on both axes, draw on a sheet of graph paper two perpendicular axes $0x$ and $0y$ for $-10 \le x \le 10$ and $-10 \le y \le 10$.

b) Given point $E(3, 2)$, $F(-1, 5)$ and the vectors $\overrightarrow{FG} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{GH} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, find the coordinates of the points G and H.

c) Draw, on the same graph, indicating clearly the vertices and their coordinates, the
   i) quadrilateral $EFGH$;
   ii) image $E_1F_1G_1H_1$ of the quadrilateral $EFGH$ under an anticlockwise rotation of $90°$ about the origin where $E \rightarrow E_1$, $F \rightarrow F_1$, $G \rightarrow G_1$ and $H \rightarrow H_1$.

d) The side E_1F_1 of the quadrilateral $E_1F_1G_1H_1$ cuts the x-axis at the point P.
   Calculate correct to one decimal place, the area of $E_1H_1G_1P_1$.

Solution

a) The graph is shown in the following figure.

b) $\overrightarrow{OG} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$ $\qquad \therefore G(0,8)$

Also $\overrightarrow{OH} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} = (3, 7)$ $\qquad \therefore H(3,7)$

c) The graph is as follows:
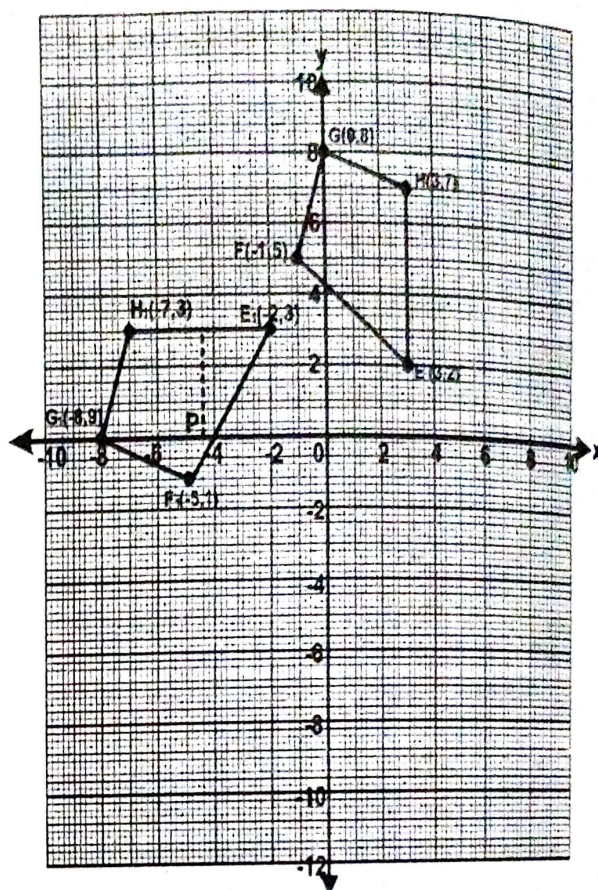
d) The length of the parallel sides are 5.0 units, 3.7 units and the height is 3.0 units.
   Area of $E_1H_1G_1P_1 =$
   $\frac{1}{2}(5.0 + 3.8)3 = 13.2$ square units

Question 13

a) Given that $P = \begin{pmatrix} y & 8 \\ 3 & 2 \end{pmatrix}$, $Q = \begin{pmatrix} -3 & -5 \\ -4 & x \end{pmatrix}$ and $PQ = R$, find the values of x, y and z

b) i) Draw on a graph paper, using a scale of 2 cm to 1 unit on both axes, the lines $x = 1$; $y = 2$; and $x + y = 5$.

   ii) Shade the region which satisfies simultaneously the inequalities $x + y \le 5$; $y \ge 2$ and $x \ge 1$

Solution

PQ = R

$$\begin{pmatrix} y & 8 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -3 & -5 \\ -4 & x \end{pmatrix} = \begin{pmatrix} -3y-32 & -5y+8x \\ -9-8 & -15-2x \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -3y-32 & -5y+8x \\ -9-8 & -15-2x \end{pmatrix} = \begin{pmatrix} -59 & -93 \\ z & -27 \end{pmatrix}$$

$\Rightarrow -3y-32=-59 \qquad \Rightarrow -3y=-59+32=-27$
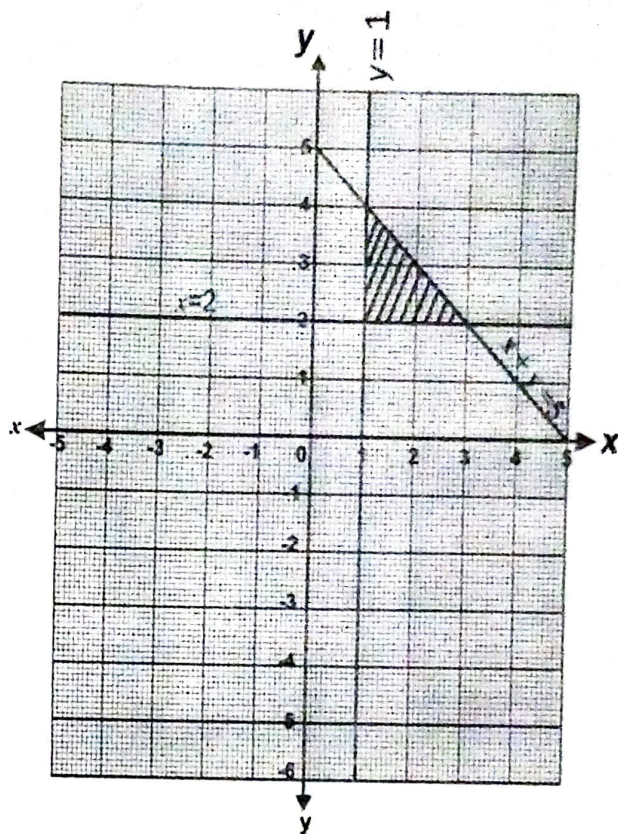
$$\therefore y = \frac{-27}{-3} = 9$$

Similarly,

$-15+2x=-27 \qquad \Rightarrow 2x=-27+15=-12$

$$\therefore x = \frac{-12}{2} = -6$$

Also $-9-8=-z \qquad \Rightarrow z=-17$

b) i) The graph is as follows:

42

NOV. 2018 OBJECTIVE TEST

1. Correct 9453×10^{-6} to 3 significant figures.
   A. 0.009
   B. 0.00945
   C. 0.00950
   D. 0.010

2. Arrange the following in ascending order of magnitude: 0.45, $\dfrac{3}{4}$ and 25 %.

   A. $\dfrac{3}{4}$, 0.45, 25%
   B. $\dfrac{3}{4}$, 25%, 0.45
   C. 0.45, 25%, $\dfrac{3}{4}$
   D. 25%, 0.45, $\dfrac{3}{4}$

3. If $\log_{10} 2 = 0.3010$ and $\log_{10} 2^y = 1.8060$, find, correct to the **nearest** whole number, the value of y.
   A. 7
   B. 6
   C. 5
   D. 4

4. Simplify: $\dfrac{\sqrt{72}}{\sqrt{48} - \sqrt{3}}$.

   A. $\dfrac{2\sqrt{6}}{3}$
   B. $2\sqrt{2}$
   C. $\dfrac{3\sqrt{2}}{2}$
   D. $\dfrac{2\sqrt{3}}{3}$

5. Which of the following describes the set $P = \{1, 2, 3, 4, 5, 6\}$?
   A. $P = \{$Prime numbers $< 7\}$
   B. $P = \{x : x$ is a positive integer $< 7\}$
   C. $P = \{$rational numbers $< 7\}$
   D. $P = \{x : x$ is a real number $< 7\}$

6. If y varies inversely as x and $y = 6$ when $x = 3$, find y when $x = 9$.
   A. 4
   B. 3
   C. 2
   D. 1

7. If the sequence x, 4, 16, y is in Geometric Progression (GP), find the ratio of $x : y$.
   A. $64 : 1$
   B. $8 : 1$
   C. $1 : 3$
   D. $1 : 64$

8. Make x the subject of the relation:
   $$E = \dfrac{kx^2}{2y} + z.$$

   A. $x = \left[\dfrac{2y(E - z)}{k}\right]^2$
   B. $x = \dfrac{2y(E + z)}{k^2}$
   C. $x = \sqrt{\dfrac{2y(E - z)}{k}}$
   D. $x = \sqrt{2yk(E + z)}$

9. If $\dfrac{1\frac{1}{2}}{2\frac{2}{3}} = \dfrac{x}{96}$, find the value of x.
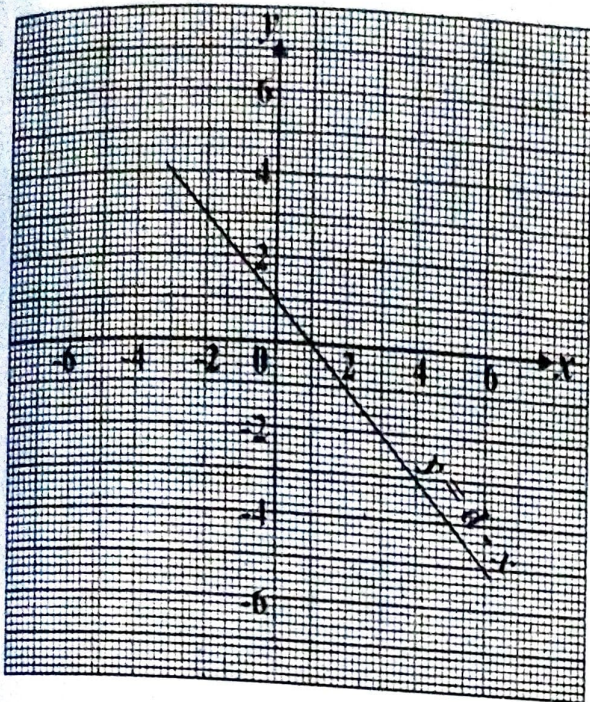   A. 42
   B. 45
   C. 54
   D. 64

10. Find the equation whose roots are $-\frac{1}{2}$ and $1\frac{1}{2}$.
    A. $2x^2 - 4x + 6 = 0$
    B. $4x^2 - 4x - 3 = 0$
    C. $2x^2 + 3x + 4 = 0$
    D. $4x^2 - 4x + 3 = 0$

11. The graph is for the relation: $y = d - x$.

Find the value of d.

A. 2   B. 1   C. 0   D. -1

12. Factorise: $p^2 q^2 - 6pqr + 9r^2$.

A. $(pq - 3r)^2$ B. $(pq - 3r)(pq + 3r)$

C. $(pq + 3r)^2$ D. $(pr + 3q)(pr - 3q)$

13. An empty rectangular tank is 250 cm long and 120 cm wide. If 180 litres of water is poured into the tank, calculate the height of the water.

A. 4.5 cm   B. 5.0 cm

C. 5.5 cm   D. 6.0 cm

14. The area of a trapezium is 49 cm². If the parallel sides are 6 cm and 7 cm long, find, correct to **one** decimal place, the distance between the parallel sides.

A. 6.5 cm   B. 6.8 cm

C. 7.4 cm   D. 7.5 cm

15. The area of a sector of a circle and the length of its arc are 231 cm² and 66 cm respectively. Calculate the radius of the circle.   [Take $\pi = \frac{22}{7}$].

A. 3.5 cm   B. 7.0 cm

C. 10.5 cm   D. 14.0 cm

16. A ladder, 10 m long, touches a side of a building at a height of 8 m. At what height would a ladder with length 12 m touch the building, if it makes the same angle with the ground?

(Assume that the ladder and building are on the same horizontal ground)

A. 10.6 m   B. 10.4 m

C. 10.0 m   D. 9.6 m

17.
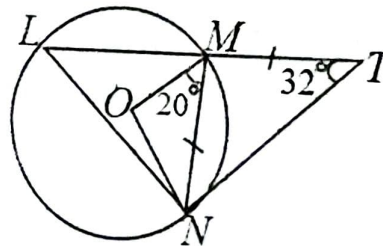
In the diagram, LMT is a straight line. If O is the centre of circle LMN, $\angle OMN = 20°$, $\angle LTN = 32°$ and $|NM| = |MT|$, find $\angle LNM$.

A. 44°   B. 46°

C. 52°   D. 76°

18. Which of these statements about an acute-angled triangle is **true**?

A. It has three equal angles.

B. It has two equal sides.

C. It has all its angles less than 90°.

D. It has one angle less than 90°.

19.

In the diagram, $SR /\!/ PQ$, $\angle SRH = 42°$, $\angle RHQ = 65°$ and $\angle HQP = y$. Find the value of y.

A. 73°   B. 65°

C. 42°   D. 23°

20.

Find the value of $\angle RPT$ in the diagram.

A. 20°   B. 40°

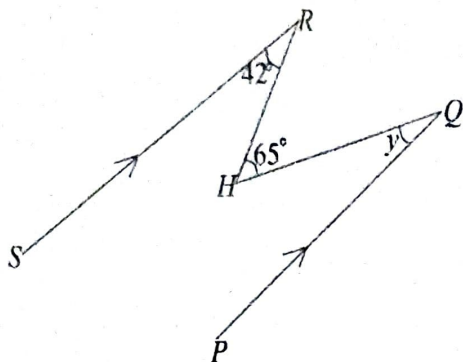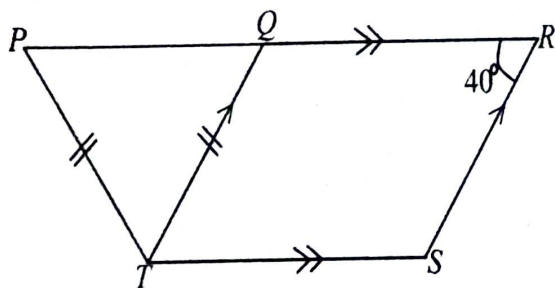C. 50°   D. 60°

21. The points $O(0, 0)$, $P(4, -1)$ and $Q(1, -4)$ are the vertices of ΔOPQ. What kind of triangle is ΔOPQ?

A. Equilateral   B. Isosceles

C. Right-angled   D. Scalene

22. Which of the following points lies on the graph of $y = x^2 + 1$?

A. $(3, 9)$

B. $(2\sqrt{2}, 8)$

C. $(3, 8)$

D. $(2\sqrt{2}, 9)$

23. A tree 8 m tall casts a shadow 10 m long. How tall is a tree on the same ground level that casts a shadow 40 m long at the same time of the day?

A. 32 m   B. 50 m

C. 80 m   D. 96 m

24. If $\cos(3x + 28°) = \sin(2x + 48°)$, $0° \le x \le 90°$, find the value of x.

A. 2.8°   B. 15.2°

C. 20.0°   D. 22.0°

25. Sammy moved 6 km due west and then 6 km on a bearing of 300°. What is the bearing of his initial position from his final position?
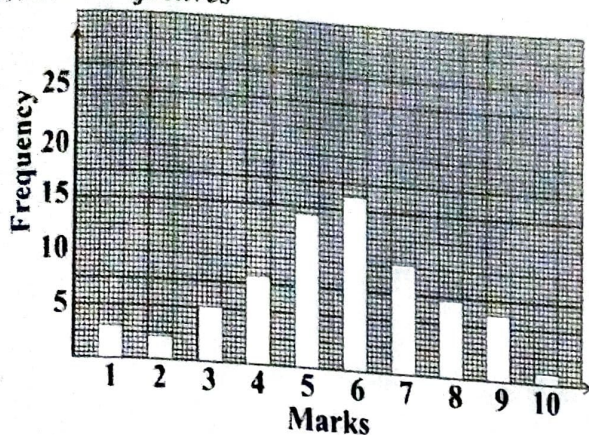
A. 105°   B. 150°

C. 165°   D. 195°

26. The table shows the distribution of the number of goals scored by a football team during a football competition.

No. of goals	1	2	3	4	5
Frequency	6	3	4	1	2

Calculate, correct to one decimal place, the mean number of the goals scored.

A. 2.3   B. 2.4

C. 2.5   D. 2.6

The bar chart represents the distribution of marks scored by students in a Mathematics test. Use the chart to answer questions 27 to 29.

27. How many students took the test?
    A. 80
    B. 72
    C. 60
    D. 52

28. If the pass mark was 5, what percentage of the students failed the test?
    A. 25 %
    B. 28 %
    C. 35 %
    D. 42 %

29. Find the probability that a student selected at random obtained the median mark.
    A. $\dfrac{16}{25}$
    B. $\dfrac{3}{5}$
    C. $\dfrac{9}{35}$
    D. $\dfrac{2}{9}$

30. Out of 20 girls in a class, 12 like Music and 15 like movies. If a girl is selected at a random from the class, what is the probability that she liked both Music and Movies?
    A. $\dfrac{3}{10}$
    B. $\dfrac{7}{20}$
    C. $\dfrac{3}{5}$
    D. $\dfrac{3}{4}$

31. The mean of the numbers 2, 5, $2x$ and 7 is not greater than 5. Find the range of values of x.
    A. $x \le 3$
    B. $x \ge 3$
    C. $x < 3$
    D. $x > 3$

32. If $3^{-x} = k$ what is 3^x?
    A. $-k$
    B. 3^k
    C. k^3
    D. $\dfrac{1}{k}$

33. To arrive on schedule, a ferry is to cover a distance of 40km at 50 km/h. If the ferry delays for 18 minutes before starting the journey, at what speed must it move so as to arrive on schedule?
    A. 70 km/h
    B. 80 km/h
    C. 90 km/h
    D. 100 km/h

34. Consider the statements:
    p: the well is wide;
    q: the well is clean;
    r: the well is deep.

    Write in symbolic form the statement "if the well is wide and clean, then it is deep".
    A. $p \wedge q \to r$
    B. $p \vee q \to r$
    C. $p \wedge q \vee r$
    D. $p \vee q \Leftrightarrow r$

35. Which of the following number lines illustrates the solution of
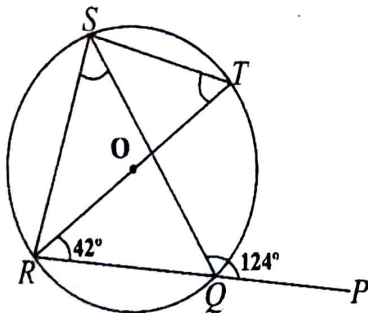 $$\frac{1}{3}(2x - 9) \ge 2(2x + 1)?$$

    A.

    B.

    C.

    D.

36.
$$
\begin{array}{r}
10101 \\
1001 \\
+***** \\
\hline
111001
\end{array}
$$

Find the missing number in the addition in base 2.

A. 10111   B. 11001

C. 11011   ᵗᵗ   D. 11010

37. A woman was y years old when she had a daughter. When the daughter was x years old, the mother was n times as old as her daughter. Express y in terms of n and x.

A. $y = n(x-1)$ B. $y = x(n-1)$

C. $y = n - x$ D. $y = x$

In the diagram, O is the centre of the circle $QRST$. $\angle QRT = 42°$ and $\angle PQS = 124°$.
Use it to answer questions **38** and **39**.

38. Find the size of $\angle RSQ$.

A. 34°   B. 48°

C. 56°   D. 76°

39. Find $\angle STR$.

A. 76°   B. 56°

C. 34°   D. 21°

40. Simplify: $\dfrac{2x+1}{2} - \dfrac{3x-7}{9} - \dfrac{5}{18}$

A. $\dfrac{2x+18}{3}$ B. $\dfrac{2x+11}{3}$

C. $\dfrac{2x+3}{3}$ D. $\dfrac{2x+1}{3}$

41. Given that one of the roots of the equation $2x^2 + (k+2)x + k = 0$ is 2, find the value of k.

A. -4 B. -2

C. -1 D. $-\frac{1}{4}$

42.
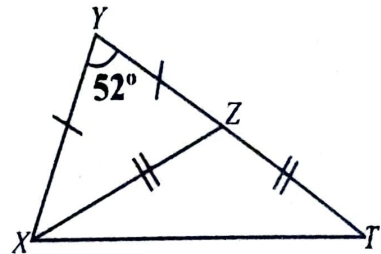
In the diagram, YT is a straight line, $|XY| = |YZ|$, $|XZ| = |ZT|$ and $\angle XYZ = 52°$
Calculate $\angle ZTX$.

A. 24°   B. 32°

C. 40°   D. 64°

43. A farmer cleared 40 % of a piece of land the first day and 60 % of the remainder the next day. What percentage of the land was remaining at the end of the second day?

A. 30 %   B. 24 %

C. 20 %   D. 15 %

44. The length of an arc of a circle is 11 cm. If it subtends an angle of $60°$ at the centre of the circle, calculate the radius of the circle. [Take $\pi = \frac{22}{7}$]

A. 7 cm

B. $7\frac{1}{2}$ cm

C. 10 cm

D. $10\frac{1}{2}$ cm

45. The mean of the sum of the squared deviations from the mean is referred to as
A. mean deviation
B. standard deviation
C. variance
D. actual mean

46. The interior angles of a pentagon are $y°$, $2x°$, $3x°$, $2x°$ and $y°$.

If $y = \dfrac{3x}{2}$, find the value of y.

A. 72

B. 81

C. 108

D. 126

47. A right pyramid is on a square base, the length of the base is equal to the vertical height of the pyramid. If the volume is $114\frac{1}{3}$ cm^3, find its height.

A. 4.9 cm

B. 7.0 cm

C. 10.1 cm

D. 18.5 cm

48. Given that $2x + 1 \equiv 4 \pmod 7$, where x is an integer. Find the least value of x.
A. 2
B. 3
C. 4
D. 5

49. Simplify: $\dfrac{2x}{1-x^2} + \dfrac{1}{1+x}$.

A. $\dfrac{1}{1-x}$

B. $\dfrac{1}{1-x^2}$

C. $\dfrac{2x+1}{1-x^2}$

D. $\dfrac{2x+1}{1-x}$

50. The circumference of a circular track is 9 km. A cyclist rides round it a number of times and stops after covering a distance of 302 km. How far is the cyclist from the starting point?

[Take $\pi = \frac{22}{7}$]

A. 6 km

B. 5 km

C. 4 km

D. 3 km

NOV. 2018 THEORY
QUESTIONS AND ANSWERS

Question 1
Solve:

a) $\frac{1}{2}(4x-6)-\frac{1}{3}(5-4x) \geq 8$.

b) the simultaneous equations:

$$\frac{3}{x}-\frac{4}{y}=\frac{1}{3},$$

$$\frac{2}{x}-\frac{5}{y}=1.$$

Solution

a) $\frac{1}{2}(4x-6)-\frac{1}{3}(5-4x) \geq 8$ LCM is 6

$3(4x-6)-2(5-4x) \geq 48$

$12x-18-10+8x \geq 48$

$12x+8x \geq 48+18+10 \Rightarrow 20x \geq 76$

$\therefore x \geq 3\frac{4}{5}$ or $x \geq \frac{19}{5}$ or $x \geq 3.8$

b) Multiplying $\frac{3}{x}-\frac{4}{y}=\frac{1}{3}$ by 5 gives:

$$\frac{15}{x}-\frac{20}{y}=\frac{5}{3} \quad ....(1)$$

Multiplying $\frac{2}{x}-\frac{5}{y}=1$ by 4 gives:

$$\frac{8}{x}-\frac{20}{y}=4 ....(2)$$

Equation (1) – (2) gives the following:

$$\left(\frac{15}{x}-\frac{8}{x}\right)+\left(-\frac{20}{y}+\frac{20}{y}\right)=\frac{5}{3}-4$$

Since the expressions in brackets have common denominators, simplifying the results gives:

$$\frac{7}{x}=\frac{-7}{3} \quad \Rightarrow \quad -7x=21$$

$$\therefore \quad x=\frac{21}{-7}=-3$$

Substituting $x=-3$ in equation (1) gives:

$\frac{15}{-3}-\frac{20}{y}=\frac{5}{3} \quad \Rightarrow \quad -5-\frac{5}{3}=\frac{20}{y}$

$\frac{-15-5}{3}=\frac{20}{y} \quad \Rightarrow \quad \frac{-20}{3}=\frac{20}{y}$

$-20y=60 \quad \Rightarrow \quad y=\frac{60}{-20}$

$\therefore y=-3$

Method 2
We may eliminate the fractions in the original equation as follows:

Let $p=\frac{1}{x}$ and $q=\frac{1}{y}$

Hence $\frac{3}{x}-\frac{4}{y}=\frac{1}{3}$ becomes:

$$3p-4q=\tfrac{1}{3}.................................(1)$$

Also $\frac{2}{x}-\frac{5}{y}=1$ becomes:

$2p-5q=1--------(2)$

(1) × 2: $6p-8q=\tfrac{2}{3}----(3)$

(2) × 3: $6p-15q=3---(4)$

(3) – (4): $-8q+15q=\tfrac{2}{3}-3$

$7q=\tfrac{-7}{3} \quad \Rightarrow 21q=-7$

$\therefore q=\frac{-7}{21}=-\frac{1}{3}$

Substituting for q in equation (2) gives:

$2p-5\left(\tfrac{-1}{3}\right)=1 \Rightarrow 2p+\tfrac{5}{3}=1$

$2p=1-\tfrac{5}{3} \quad \Rightarrow 2p=\tfrac{-2}{3}$

$\therefore p=\tfrac{-2}{3}\div 2=\tfrac{-1}{3}$

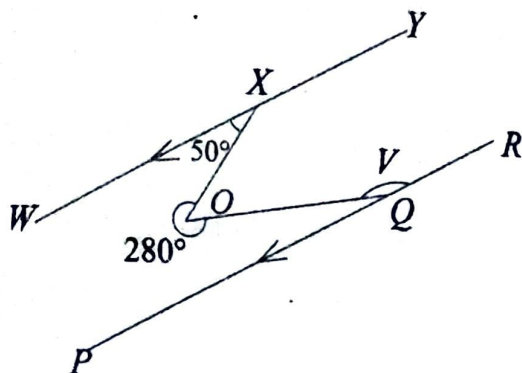But $p=\frac{1}{x} \quad \Rightarrow \quad \tfrac{-1}{3}=\tfrac{1}{x} \quad \Rightarrow \quad -x=3$

$\therefore x=-3$ and $y=-3$

Nov. 2018 *Theory*

Question 2

a) A woman spent $\frac{1}{6}$ of her monthly salary on foodstuffs $\frac{1}{3}$ on drugs, $\frac{1}{4}$ on utility bills and had GH¢ 285.00 left. Calculate her monthly salary.

b)

In the diagram $WY \parallel PR$, $\angle WXO = 50°$ reflex $\angle XOQ = 280°$ and $\angle OQR = V$. Find the value of V.

Solution

a) Let monthly salary $= x$

Fraction spent $= \frac{1}{6} + \frac{1}{3} + \frac{1}{4} = \frac{3}{4}$

Fraction left $= 1 - \frac{3}{4} = \frac{1}{4}$

This means that $\frac{1}{4}x = 285$

So $x = 285 \times 4$

$\therefore x = GH\, ¢1,140.00$

b) **Method 1**
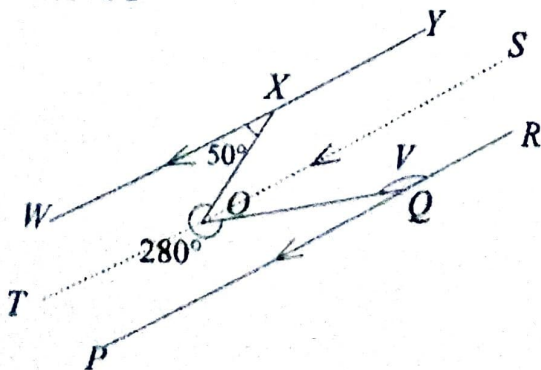
Nov. 2018 *Theory*

$\angle WXO = \angle XOS = 50°$

$\angle XOQ = \angle SOQ = 80° - 50° = 30°$

$\therefore V = 180° = 150°$

Method 2
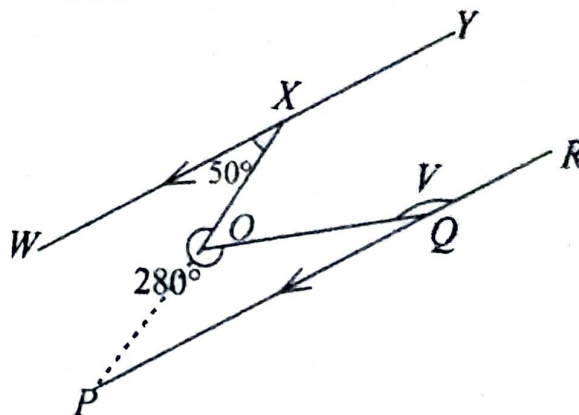
$\angle XOQ = 360° - 280° = 80°$

$80° + \angle POQ = 180°$

$\angle POQ = 100°$

$\angle XPQ = 50°$

$V = 50° + 100°$

$V = 150°$

Question 3

From two points P and Q, 15 m apart and on the same horizontal line as the foot of a tower, the angles of elevation of the top of the tower are 35° and 45° respectively.

If P and Q are on the side of the tower,

a) represent the information in a diagram.

b) find, correct to the **nearest metre**, the height of the tower.

591

Solution

a) The following is an illustration of the information:
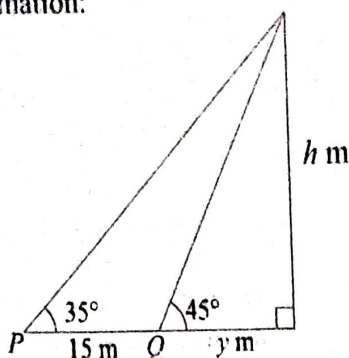
b) $\tan 35^0 = \dfrac{h}{15+y}$

$h = (15+y)\tan 35^\circ$ --------(1)

$45^\circ = \dfrac{h}{y}$

$h = y\tan 45^\circ$ -----------(2)

Equating (1) and (2)

$(15+y)\tan 35^\circ = y\tan 45^\circ$

$(15+y)0.7002 = y$

$0.2998y = 10.503$

$y = 35.0334$ m

Substituting for y in equation (2),

$h = 35.0334$ m

Height of the tower ≈ 35 m, correct to the **nearest** metre.

Question 4

a) The area of a sector of a circle of radius 12 cm, is $132\,\text{cm}^2$. If the sector is folded such that its straight edges coincide to form a cone, find the radius of the base of the cone. [Take $\pi = \frac{22}{7}$]

b) A circle centre O has radius 5 cm. A chord PQ of the circle is 6 cm long. Calculate:
   i) the distance of the chord from the centre O;
   ii) angle POQ

Solution

a) Let R = radius of the cone;
   r = radius of the cone;

The following are the sector and the cone formed:
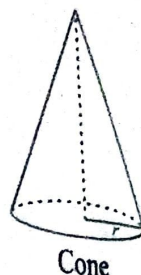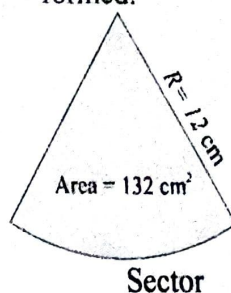
Sector   Cone

Step 1: Finding the angle of the sector

Area of sector $= \dfrac{\text{angle of sector}}{360^0} \times \pi R^2$

$132 = \dfrac{x}{360^0} \times \dfrac{22}{7} \times 12 \times 12 \qquad 132 = x \times \dfrac{44}{5}$

where x = angle of sector

$\therefore\ x = \dfrac{35 \times 132}{44} = 105^\circ$

Step 2: Finding the radius of the cone

Length of arc of sector=Circumference of base of cone

$2\pi R = \dfrac{x}{360^0} \times 2\pi r \qquad\qquad \Rightarrow\ r = \dfrac{x}{360^0} \times R$

$\therefore\ r = \dfrac{105^\circ}{360^\circ} \times 12\ = 3\tfrac{1}{2}$ cm $= 3.5$ cm

Hence the radius of cone is 3.5 cm

b) i) Let x cm be the distance of the chord from the centre, as illustrated below:
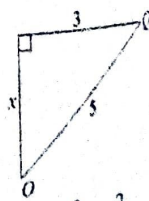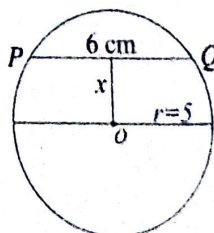
From the diagram, $\qquad x^2 + 3^2 = 5^2$

$x^2 = 5^2 - 3^2 = 25 - 9 = 16$

$$\therefore x = \sqrt{16} = 4 \text{ cm}$$

ii) Let θ be $\angle POQ$

So $\theta = 2 \times \alpha$

$$\sin \alpha = \frac{3}{5} \qquad \sin \alpha = 0.6$$

$$\alpha = \sin^{-1}(0.6) = 36.87°$$

But $\angle POQ = 2 \times \alpha = 2 \times 36.87°$

$$\therefore \text{ Angle } POQ = 73.74°$$

Question 5

The probabilities that Ade, Kofi and Fati will pass an examination are $\frac{2}{3}$, $\frac{5}{8}$ and $\frac{3}{8}$ respectively. Find the probability that:
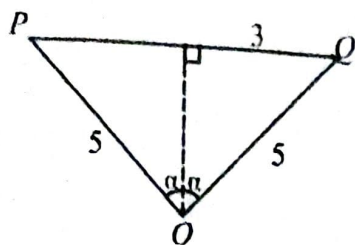a) the three of them will pass the examination.
b) none of them will pass the examination.
c) Ade and Kofi only will pass the examination.

Solution

a) $P(A) = \frac{2}{5} \qquad \Rightarrow \qquad P(\bar{A}) = \frac{1}{3}$

$P(K) = \frac{5}{8} \qquad \Rightarrow \qquad P(\bar{K}) = \frac{3}{8}$

$P(F) = \frac{3}{4} \qquad \Rightarrow \qquad P(\bar{F}) = \frac{1}{4}$

$P \text{ (three of them will pass)} = \frac{2}{3} \times \frac{5}{8} \times \frac{3}{4}$

$$= \frac{5}{16} \text{ (or } 0.3125)$$

b) $P \text{ (none of them will pass)} = \frac{1}{3} \times \frac{3}{8} \times \frac{1}{4}$

$$= \frac{1}{32} \text{ (or } 0.3125)$$

c) $P \text{ (Ade and Kofi only will pass)}$
$= \frac{2}{3} \times \frac{5}{8} \times \frac{1}{4}$

$= \frac{5}{48} \text{ (or } 0.1042)$

Question 6

a) A housing estate consists of 100 houses each rented at GH¢150.00 per month and 108 flats each rented at GH¢110.00 per month. If all were rented out in a year,
i) find the total annual rent collected;
ii) calculate the half year tax, if income tax paid on rent is 8 % per annum.

b) Two cyclists Musa and Amandu left point P at the same time in opposite directions. If their speeds are 8 km/h and 12 km/h respectively;
i) how long will it take them to be 40 km apart?
ii) calculate the distance covered by Musa within the time in b(i).

Solution

a) i) Amount collected for 100 houses
$= 100 \times \text{GH¢}150.00$
$= \text{GH¢ } 15,000.00$

Amount of rent collected
$= \text{GH¢}15,000.00 \times 12$
$= \text{GH¢ } 180,000.00$

Amount collected for 108 houses
$= 108 \times \text{GH¢}110.00$
$= \text{GH¢ } 11,880.00$

Annual rent collected
$= \text{GH¢}11,880.00 \times 12$
$= \text{GH¢ } 142,560.00$

Total annual rent collected
$= \text{GH¢}180,000 - 142,560$
$= \text{GH¢ } 322,560.00$

ii) Income tax paid on rent
$= \frac{8}{100} \times 322.560 = 25804.80$

Half year tax $= \frac{1}{8} \times 25804.80$

$$= \text{GH¢}12,902.40$$

b) i) Let t hours be the time taken

Distance by Musa $= 8t$

Distance by Amandu $= 12t$

$8t + 12t = 40$

$20t = 40$

$t = 2$ hours

iii) Distanced covered by Musa

$= 8 \times 2$

$= 16$ km

Question 7

a) Solve the equation $\dfrac{2x+3}{x-1} = \dfrac{5x-3}{3-x}$

b)
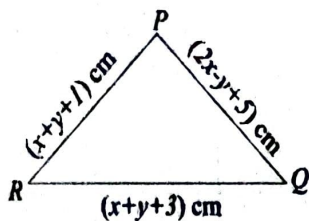
In the diagram, RQ is the base of the isosceles triangle PQR. If the perimeter of the triangle is 20 cm, find the values of x and y.

Solution

a) $\dfrac{2x+3}{x-1} = \dfrac{5x-3}{3-x}$

$(2x+3)(3-x)=(5x-3)(x-1)$

$6x-2x^2+9-3x=5x^2-5x-3x+3$

$7x^2-11x-6=0$

$(7x+3)(x-2)=0$

$x = 2$ or $\frac{-3}{7}$

b) Since $|PQ|=|PR|$,

$x+y+1=2x-y+5$

$2y-x=4$ -(1)

Since the perimeter is 20 cm, we have:

$x+y+1+2x-y+5+x+y+3=20$

$4x+y=11$ - - - - - - - - - - - - - - - - - - -(2)

From equation (1),

$x = 2y-4$ -(3)

Substituting for x in equation (2),

$4(2y-4)+y=11$ $8y-16+y=11$

$9y = 27$ $\therefore$ $y = 3$

Substituting for y in equation (3),

$x = 2(3)-4=2$

Question 8

a) Draw the table of values for $y = 2\cos x$,

$0° \le x \le 360°$ using intervals of $60°$.

b) Using scales of 2cm to $60°$ on the x-axis and 2 cm to 1 unit to on the y-axis, draw the graph of $y = 3 - 2\cos x$,

for $0° \le x \le 360°$

c) Use the graph to find the:

   i) maximum point of the graph;

   ii) values of x for which $\cos x = 0.25$.

Solution

a) The table is as follows:

x	$0°$	$60°$	$120°$	$180°$	$240°$	$300°$	$360°$
y	1	2	3	4	5	2	1

b) The graph is as follows:
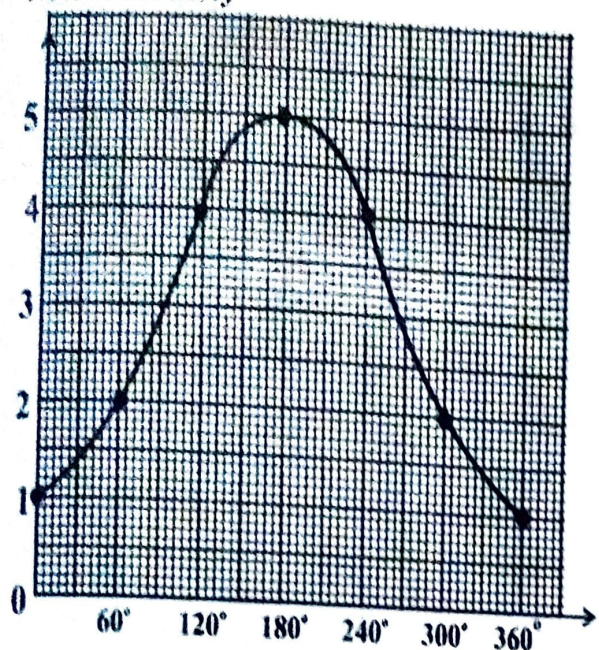
c) i) The maximum point of the graph is (x, y)
$$= (180°, 5)$$
$$\cos x = 0.25 \Rightarrow \cos x = \tfrac{1}{2}$$
But $2 \cos x = 3 - y$
So that $3 - y = \tfrac{1}{2}$
Therefore, $y = 2.5$
The values of x for which $\cos x = 0.25$ are
$x = 72°$ or $x = 282°$ (4 v.d.)

Question 8

The table shows the frequency distribution of marks scored by 100 candidates in an examination.

Marks (%)	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99
Frequency	2	7	8	13	24	30	6	5	3	2

a) Construct a cumulative frequency table.
b) Draw a cumulative frequency curve.
c) Use the curve to estimate the:
   i) percentage of candidates who passed the examination, if the pass mark is -;
   ii) **lowest** mark for a scholarship if the **best 5 %** of the candidates qualified for the scholarship.

Solution

a) The following is the cumulative frequency table:

Marks (x)	Frequency	Cumulative Frequency
Less than 9.5	2	2
Less than 19.5	7	9
Less than 29.5	8	17
Less than 39.5	13	30
Less than 49.5	24	54
Less than 59.5	30	84
Less than 69.5	6	90
Less than 79.5	5	95
Less than 89.5	3	98
Less than 99.5	2	100

b) The following is the cumulative frequency curve:
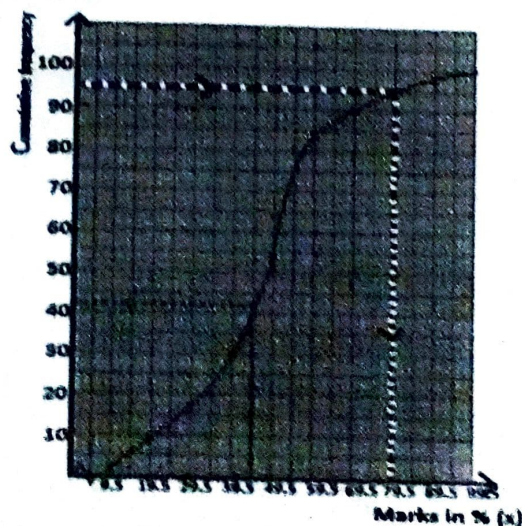
c) i) From the graph, 45.5 % corresponds to 42 on the vertical axis.

$100 - 42 = 58$

∴ The percentage of candidates that passed is 58%

ii) $100\% - 5\% = 95\%$

From the graph, 95 corresponds to 79.5 on the horizontal axis.

∴ The lowest mark for the scholarship if the best 5% of the candidates qualified is 79.5%
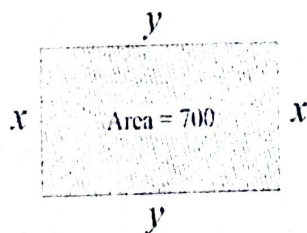
Question 10

The area of a rectangular farmland is $7,200 \text{ m}^2$ while its perimeter is 360 m. Calculate, the:

a) dimensions of the farmland;

b) cost of clearing the farmland at ₦8.50 per square metre, leaving a margin of 2 m wide along the longer sides.

c) percentage of the farmland **not** cleared.

Solution

a) Let x be the width and y be the length
The farmland is illustrated as follows:

$$xy = 7,200 \dots\dots\dots\dots (1)$$
$$2(x + y) = 360$$
$$x + y = 180 \dots\dots\dots\dots (2)$$

From equation (2), $y = 180 - x \dots\dots (3)$

Substituting for y in equation (1) gives:

$$x(180 - x) = 7200 \qquad 180x - x^2 = 7200$$

$$180x - x^2 - 7200 = 0$$

$$(x - 60)(x - 120) = 0$$

∴ $x = 60$ or 120

But $y = 180 - x$

When $x = 60$, $y = 180 - 60 = 120$

The dimensions of the farmland is 120 m by 60 m.

b) Leaving a margin of 2 m along the longer sides, the new area becomes

$$7200 - 2(2 \times 120)$$

Cost of clearing $= 6720 \times ₦ 8.50$ per square metre $= ₦57,120.00$

c) Percentage of the farmland not cleared

$$= \frac{400}{7200} \times 100 \%$$

$$= 6\frac{2}{3}\% = 6.6667 \%$$

Question 11

a) Evaluate $\int_{1}^{3}\left(\frac{3 + x^3 + 2x^4}{x^2}\right) dx.$

b) Using the completing the square method, solve the equation: $4x^2 - 8x - 5 = 0$.

Solution

a) Simplifying the expression in brackets gives us the following:

$$\int_{1}^{3}\left(\frac{3 + x^3 + 2x^4}{x^2}\right) dx = \int_{1}^{3}\left(\frac{3}{x^2} + x + 2x^2\right) dx$$

$$= \left[\frac{-3}{x} + \frac{x^2}{2} + \frac{2x^3}{3}\right]_{1}^{3}$$

$$= \left[\frac{-3}{3} + \frac{(3)^2}{2} + \frac{2(3)^3}{3}\right] - \left[\frac{-3}{1} + \frac{(1)^2}{2} + \frac{2(1)^3}{3}\right]$$

$$= \left(-1 + \frac{9}{2} + 18\right) - \left(-3 + \frac{1}{2} + \frac{2}{3}\right)$$

$$= \frac{43}{2} - \left(-\frac{11}{6}\right) = \frac{43}{2} + \frac{11}{6} = \frac{70}{3}$$

$$= 23\frac{1}{3} = (23.3333)$$

b) $4x^2 - 8x - 5 = 0$

Dividing through by 4 gives:

$x^2 - 2x - \frac{5}{4} = 0 \qquad \Rightarrow \quad x^2 - 2x = \frac{5}{4}$

Adding half the coefficient of x to both sides gives us:

$x^2 - 2x + (1)^2 = \frac{5}{4} + (1)^2$

$(x-1)^2 = \frac{5}{4} + 1$ $\qquad (x-1)^2 = \frac{9}{4}$

$(x-1) = \pm \sqrt{\left(\frac{9}{4}\right)}$

$(x-1) = \pm \frac{3}{2} \qquad\qquad x = 1 \pm \frac{3}{2}$

Either $\quad x = 1 + \frac{3}{2} = \frac{5}{2}$

Or $\qquad x = 1 - \frac{3}{2} = \frac{-1}{2}$

Question 12

a) **M** (3, 7) and **N** (15, −3) are two points in the coordinate plane. **P** is a point on **MN** such that **MP** = k**MN**. Find the coordinates of **P** when $k = \frac{3}{2}$

b) The point **P** has position vector $\begin{pmatrix} 100 \\ 0 \end{pmatrix}$ and

$T = \begin{pmatrix} 0.28 & 0.96 \\ 0.96 & 0.28 \end{pmatrix}$

i) Find the images **P'** of P under **T**.

ii) Using 2 cm to 20 units on both axes, plot the position vectors of **P** and **P'**

iii) Draw a straight line **L**, bisecting the angle P'OP.

iv) Measure $\angle$P'OL.

Solution

a) Let (x, y) be the coordinates of P

MP = k**MN**

$\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \frac{-3}{2} \begin{pmatrix} 12 \\ -10 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} -18 \\ 15 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -18 \\ 15 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -15 \\ 22 \end{pmatrix}$

$\therefore$ The coordinates of P$(-15, 22)$

b) i) The image of **P** under T is:

$P^1 = \begin{pmatrix} 0.28 & 0.96 \\ 0.96 & -0.28 \end{pmatrix} \begin{pmatrix} 100 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 28 \\ 96 \end{pmatrix}$

ii) The graph is as follows:
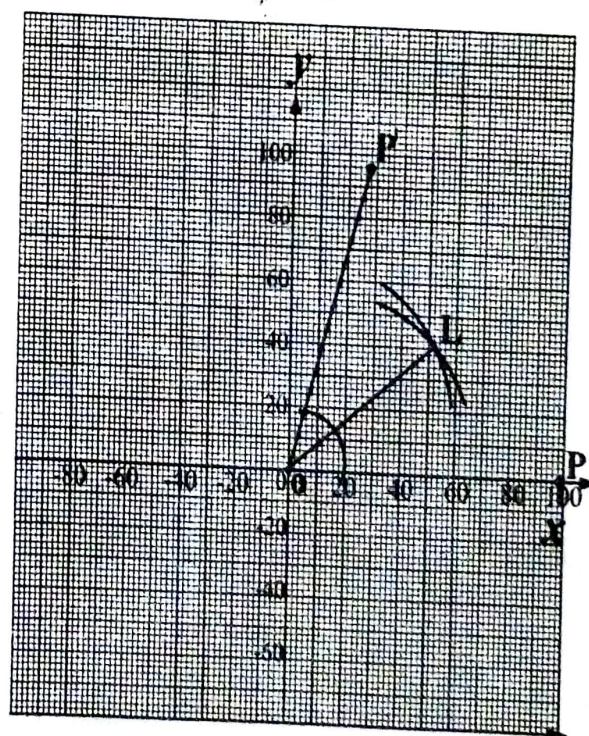
iii) Refer to the graph.

iv) $\angle P^1OL = 37°$

Question 13

a)
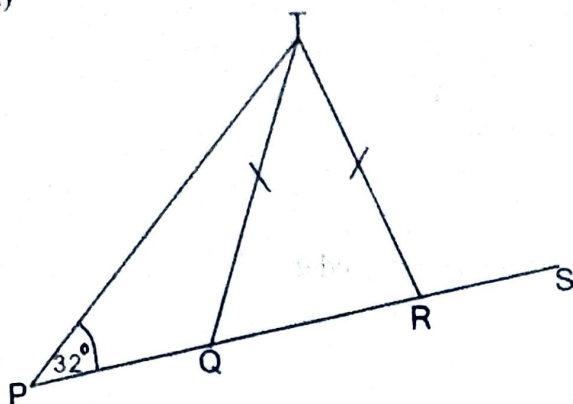
In the diagram, $\angle TPQ = 32°$ and $|PQ| = |QT| = |TR|$. Calculate $\angle TRS$.

b)
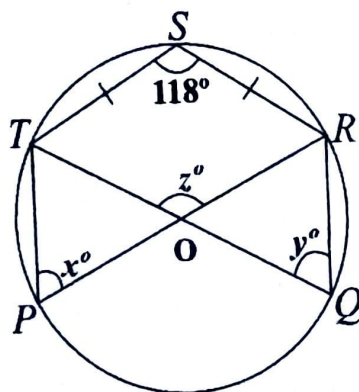
In the diagram; O is the centre of the circle . $|TS| = |SR|$, $\angle TPR = x°$, $\angle TQR = y°$, $\angle TOR = z°$ and $\angle TSR = 118°$

i) Find the relationship between x, y and z.
ii) Calculate $\angle STP$

Solution

a) $\angle TQR = 32° + 32° = 64°$ (exterior $\angle$ of a triangle)

$\angle TRQ = \angle TQR = 64°$

$\angle TRS = 180° - 64° = 116°$

b) i) $\angle TOR = 2\angle TPO$

$z = 2y$ .... (1)

$\angle TOR = Z\angle RQO$

$z = 2x$ .... (2)

Adding equations (1) and (2)

$2z = 2(x + y)$

Therefore, $z = x + y$.

$180° - 118°$

ii) $\angle STR = \dfrac{180° - 118°}{2}$

$= 31°$

Since $\overline{PR}$ is a diameter, then,

$\angle PTR = 90°$

$\angle STP = 31° + 90°$

$= 121°$

43 JULY 2019 OBJECTIVE TEST

1. Express, correct to **three** significant figures, 0.003597.

A. 0.00359

B. 0.00360

C. 0.004

D. 0.359

2. Evaluate $(0.064)^{-\frac{1}{3}}$.

A. $-\dfrac{5}{2}$

B. $-\dfrac{2}{5}$

C. $\dfrac{2}{5}$

D. $\dfrac{5}{2}$

3. Solve $\dfrac{y+1}{2} - \dfrac{2y-1}{3} = 4$

A. $y = 29$

B. $y = -29$

C. $y = -19$

D. $y = 19$

4. Simplify, correct to **three** significant figures, $(27.63)^2 - (12.37)^2$.

A. 610

B. 611

C. 612

D. 614

5. If $7 + y = 4(\text{mod } 8)$, find the least value of y, $10 \le y \le 30$.

A. 21

B. 19

C. 13

D. 11

6. If $T = \{\text{prime numbers}\}$ and $M = \{\text{odd numbers}\}$ are subsets of $\mu = \{x : 0 < x \le 10\}$, and x is an integer, find $(T' \cap M')$.

A. $\{1, 2, 3, 5, 7, 8, 9\}$

B. $\{1, 2, 4, 6, 8, 10\}$

C. $\{1, 4, 6, 8, 10\}$

D. $\{4, 6, 8, 10\}$

7. Evaluate $\dfrac{\log_3 9 - \log_2 8}{\log_3 9}$.

A. $-\dfrac{1}{2}$

B. $\dfrac{1}{3}$

C. $\dfrac{1}{2}$

D. $-\dfrac{1}{3}$

8. If $23_y = 1111_{two}$, find the value of y.

A. 7

B. 6

C. 5

D. 4

9. If 6, p and 14 are consecutive terms in Arithmetic Progression (A.P), find the value of p.

A. 8

B. 6

C. 10

D. 9

10. Evaluate $2\sqrt{28} - 3\sqrt{50} + \sqrt{72}$

A. $4\sqrt{7} + \sqrt{2}$

B. $4\sqrt{7} - 9\sqrt{2}$

C. $4\sqrt{7} - 11\sqrt{2}$

D. $4\sqrt{7} - 21\sqrt{2}$

11. If $m : n = 2 : 1$, evaluate: $\dfrac{3m^2 - 2n^2}{m^2 + mn}$.

A. $\dfrac{3}{5}$

B. $\dfrac{3}{4}$

C. $\dfrac{5}{3}$

D. $\dfrac{4}{3}$

12. H varies directly as p and inversely as the square of y. If $H = 1, p = 8$ and $y = 2$, find H in terms of p and y?

A. $H = \dfrac{p}{y^2}$

B. $H = \dfrac{p}{2y^2}$

C. $H = \dfrac{2p}{y^2}$

D. $H = \dfrac{p}{4y^2}$

13. Solve $4x^2 - 16x + 15 = 0$.

A. $x = -1\frac{1}{2}$ or $-2\frac{1}{2}$

B. $x = 1\frac{1}{2}$ or $-1\frac{1}{2}$

C. $x = 1\frac{1}{2}$ or $2\frac{1}{2}$

D. $x = 1\frac{1}{2}$ or $-2\frac{1}{2}$

14. Evaluate $\dfrac{0.42 \div 2.5}{0.5 \times 2.05}$, leaving the answer in standard form.

A. 1.639×10^{-2} B. 1.639×10^{-1}

C. 1.639×10^{1} D. 1.639×10^{2}

15. Simplify: $\log_{10} 6 - 3\log_{10} 3 + \dfrac{2}{23}\log_{10} 27$

A. $2\log_{10} 3$ B. $\log_{10} 3$

C. $\log_{10} 2$ D. $3\log_{10} 2$

16. Bala sold an article for ₦6,900.00 and made a profit of 15%. Calculate his percentage profit if he had sold it for ₦6,600.00.

A. 13 %   B. 12%

C. 10 %   D. 5 %

17. If $3p = 4q$ and $9p = 8q - 12$, find the value of pq.

A. –12   B. –7

C. 7   D. 12

18. If $(0.25)^y = 32$, find the value of y.

A. $-\dfrac{5}{2}$ B. $-\dfrac{3}{2}$

C. $\dfrac{3}{2}$ D. $\dfrac{5}{2}$

19. There are 8 boys and 4 girls in a lift. What is the probability that the first person who steps out of the lift will be a boy?

A. $\dfrac{1}{4}$ B. $\dfrac{2}{3}$

C. $\dfrac{1}{3}$ D. $\dfrac{3}{4}$

20. Simplify: $\dfrac{x^2 - 5x - 14}{x^2 - 9x + 14}$.

A. $\dfrac{x+2}{x-2}$ B. $\dfrac{x-2}{x+4}$

C. $\dfrac{x+7}{x-7}$ D. $\dfrac{x-7}{x+7}$

21. Which of these values would make $\dfrac{3p-1}{p^2 - p}$ undefined?

A. -1 B. $-\dfrac{1}{3}$

C. $\dfrac{1}{3}$ D. 1

22. The total surface area of a solid cylinder is 165 cm^2. If the base diameter is 7 cm, calculate its height. [Take $\pi = \frac{22}{7}$]

A. 2.0 cm

B. 4.0 cm

C. 4.5 cm

D. 7.5 cm

23. If $2^a = \sqrt{64}$ and $\dfrac{b}{a} = 3$, evaluate $a^2 + b^2$.
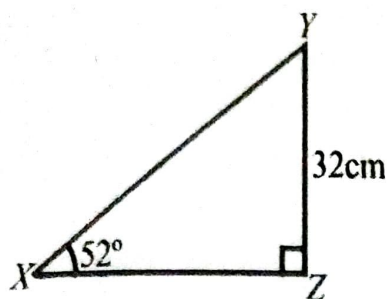
A. 48   B. 90

C. 160   D. 250

24.

NOT DRAWN TO SCALE

In $\triangle XYZ$, $|YZ| = 32$ cm, $\angle YXZ = 52°$ and $\angle XZY = 90°$. Find, correct to the **nearest** centimetre, $|XZ|$.

A. 13 cm   B. 20 cm
C. 25 cm   D. 31 cm

25. If $\log_x 2 = 0.3$, evaluate $\log_x 8$.

A. 0.6   B. 0.9
C. 1.2   D. 2.4

26. An arc subtends an angle of 72° at the centre of a circle. Find the length of the arc if the radius of the circle is 3.5 cm. [Take $\pi = \frac{22}{7}$]

A. 2.2 cm   B. 4.4 cm
C. 8.8 cm   D. 6.6 cm

27. Make b the subject of the relation:

$$lb = \frac{1}{2}(a+b)h$$

A. $\dfrac{al}{2-h}$ B. $\dfrac{al}{2l-h}$

C. $\dfrac{2l-h}{al}$ D. $\dfrac{ah}{2l-h}$

28. Eric sold his house through an agent who charged 8 % commission on the selling price. If Eric received $117,760.00 after the sale, what was the selling price of the house?

A. $ 120,000.00   B. $ 125,000.00
C. $ 128,000.00   D. $ 130,000.00

29. Find the angle which an arc of length 22 cm subtends at the centre of a circle of radius 15 cm. [Take $\pi = \frac{22}{7}$]

A. 156°   B. 96°
C. 84°   D. 70°

30. A rectangular board has length 15 cm and width x cm. If the sides are doubled, find its new area?

A. $15x$ cm^2 B. $30x$ cm^2
C. $45x$ cm^2 D. $60x$ cm^2

31.
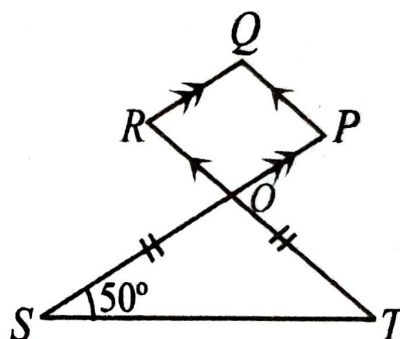
NOT DRAWN TO SCALE

In the diagram, POS and ROT are straight lines. $OPQR$ is a parallelogram, $|OS| = |OT|$ and $\angle OST = 50°$. Calculate the value of $\angle OPQ$.

A. 160°   B. 140°
C. 120°   D. 100°

32. Factorize completely:
$(2x + 2y)(x - y) + (2x - 2y)(x + y)$.

A. $2(x-y)$ B. $2(x-y)(x+y)$
C. $4(x-y)$ D. $4(x-y)(x+y)$

33. The interior angles of a polygon are $3x°$, $2x°$, $4x°$, $3x°$ and $6x°$. Find the size of the **smallest** angle of the polygon.

A. 30°   B. 40°
C. 60°   D. 80°

34. A box contains 2 white and 3 blue identical balls. If two balls are picked at random from the box, one after the other with replacement, what is the probability that they are of different colours?

A. $\dfrac{12}{25}$ B. $\dfrac{7}{20}$

C. $\dfrac{3}{5}$ D. $\dfrac{2}{3}$

35. Find the equation of a straight line passing through the point $(1, -5)$ and having gradient of $\dfrac{3}{4}$.

A. $3x - 4y - 23 = 0$
B. $3x - 4y + 23 = 0$
C. $3x + 4y + 23 = 0$
D. $3x + 4y - 23 = 0$

36. The foot of a ladder is 6 m from the base of an electric pole. The top of the ladder rests against the pole at a point 8 m above the ground. How long is the ladder?
A. 7 m B. 10 m
C. 12 m D. 14 m

37. If $\tan x = \dfrac{3}{4}$, $0 < x < 90$, evaluate $\dfrac{\cos x}{2\sin x}$

A. $\dfrac{2}{3}$ B. $\dfrac{4}{3}$

C. $\dfrac{3}{4}$ D. $\dfrac{8}{3}$

38. From the top of a vertical cliff 20 m high, a boat at sea can be sighted 75 m away and on the same horizontal position as the foot of the cliff. Calculate, correct to the near   ee, the angle of depression of the b   n the top of the cliff.
A. 15° B. 16°
C. 75° D. 56°

39.
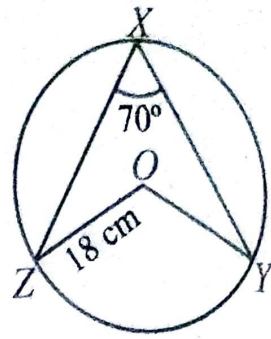
NOT DRAWN TO SCALE

In the diagram, O is the centre of the circle with radius 18 cm. If the angle $\angle ZXY = 70°$, calculate the length of arc ZY. [Take $\pi = \dfrac{22}{7}$]
A. 80 cm B. 44 cm
C. 22 cm D. 11 cm
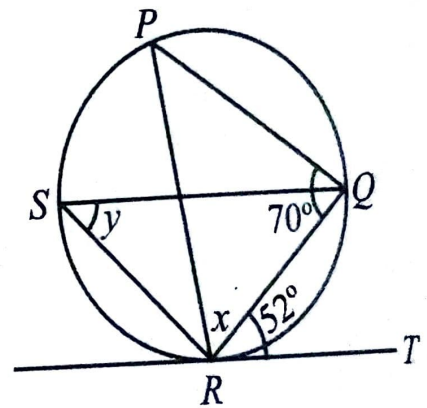
NOT DRAWN TO SCALE

In the diagram, RT is a tangent to the circle at R, $\angle PQR = 70°$, $\angle QRT = 52°$, $\angle QSR = y$ and $\angle PRQ = x$. Use the diagram to answer questions 40 and 41.

40. Find the value of y.
A. 18° B. 52°
C. 60° D. 60°

41. Calculate the value of x.
A. 48° B. 55°
C. 58° D. 70°

42. Calculate the variance of 2, 4, 7, 8 and 9.
    A. 2.6
    B. 3.5
    C. 6.8
    D. 7.2

43. The fourth term of an Arithmetic Progression (A.P) is 37 and the first term is -20. Find the common difference.
    A. 17
    B. 19
    C. 57
    D. 63
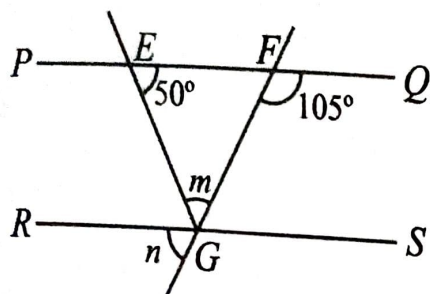
NOT DRAWN TO SCALE

In the diagram, $\overline{PQ}$ is parallel to $\overline{RS}$, $\angle QFG = 105°$ and $\angle FEG = 50°$. Use the diagram to answer questions 44 and 45.

44. Find the value of m.
    A. 55°
    B. 75°
    C. 105°
    D. 130°

45. Find the value of n.
    A. 130°
    B. 75°
    C. 55°
    D. 40°

46. A box contains 5 red, 6 green and 7 yellow pencils of the same size. What is the probability of picking a **green** pencil at random?
    A. $\dfrac{1}{2}$
    B. $\dfrac{1}{3}$
    C. $\dfrac{1}{4}$
    D. $\dfrac{1}{6}$

47.
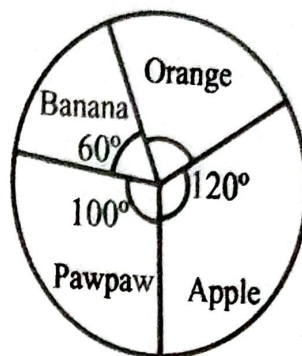
NOT DRAWN TO SCALE

The pie chart represents fruits on display in a grocery shop. If there are 60 oranges on display, how many apples are there?
    A. 40
    B. 80
    C. 90
    D. 70

The following are scores obtained by some students in a test:

8   18   10   14   18   11   13

14   13   17   15   8   16   13

Use this information to answer questions 48 to 50.

48. Find the mode of the distribution.
    A. 8
    B. 13
    C. 14
    D. 18

49. Find the median score.
    A. 13.0
    B. 13.5
    C. 14.0
    D. 14.5

50. How many students scored above the mean score?
    A. 7
    B. 8
    C. 9
    D. 10

JULY 2019 THEORY QUESTIONS AND ANSWERS

Question 1

a) Without using mathematical tables or calculators, evaluate $\dfrac{0.015 \times 0.567}{0.05 \times 0.189}$, leaving the answer in standard form.

b) If $\dfrac{5y-x}{8y+3x} = \dfrac{1}{5}$, find, correct to **two decimal** places, the value of $\dfrac{x}{y}$.

Solution

a) $\dfrac{0.015 \times 0.567}{0.05 \times 0.189} = \dfrac{15 \times 10^{-3} \times 567 \times 10^{-3}}{5 \times 10^{-2} \times 189 \times 10^{-3}}$

$= \dfrac{3 \times 3 \times 10^{-6}}{1 \times 10^{-5}} = 9 \times 10^{-6+5} = 9 \times 10^{-1}$

b) $\dfrac{5y-x}{8y+3x} = \dfrac{1}{5}$ gives $5(5y-x) = 8y+3x$

$25y - 5x = 8y + 3x \qquad 25y - 8y = 3x + 5x$

$17y = 8x \qquad\qquad \dfrac{x}{y} = \dfrac{17}{8}$

$\therefore \dfrac{x}{y} = 2.125 \approx 2.13 \quad$ (2 d.p)

Question 2

a) Z varies directly as x and inversely as **twice** the cube root of y. If $Z = 8$, when $x = 4$ and $y = \dfrac{1}{8}$, find the relation for y in terms of x and Z.

b) Factorize completely:

$4b^2 - ab + (a+9b)^2 - a^2$.

Solution

a) $Z \ \alpha \ \dfrac{x}{2(\sqrt[3]{y})} \qquad\qquad Z = \dfrac{kx}{2(\sqrt[3]{y})}$

$Z = 8, \ x = 4, \ y = \dfrac{1}{8} \qquad 8 = \dfrac{4k}{2(\sqrt[3]{\frac{1}{8}})}$

$(8)(2)(\tfrac{1}{2}) = 4k \qquad\qquad \therefore k = 2$

This gives: $\quad Z = \dfrac{2x}{2(\sqrt[3]{y})} \qquad \sqrt[3]{y} = \dfrac{x}{Z}$

$\therefore \ y = \left(\dfrac{x}{Z}\right)^3$

b) $4b^2 - ab + (a+9b)^2 - a^2$

$= 4b^2 - ab + a^2 + 18ab + 81b^2 - a^2$

$= 4b^2 + 81b^2 - ab + 18ab$

$= 85b^2 + 17ab$

$= 17b(5b + a)$

Question 3

a) Solve $\dfrac{5x-7}{6} + \dfrac{2x-3}{4} = -\dfrac{2}{3}$.

b) Evaluate $\dfrac{\sqrt{28} + \sqrt{343}}{2\sqrt{63}} + \dfrac{5}{3}$.

Solution

a) $\dfrac{5x-7}{6} - \dfrac{2x-3}{4} = \dfrac{-2}{3}$

Multiplying through by 12 gives:

$2(5x-7) + 3(2x-3) = 4(-2)$

$10x - 14 + 6x - 9 = -8$

$16x = 15 \qquad\qquad \therefore x = \dfrac{15}{16}$

b) $\dfrac{\sqrt{28} + \sqrt{343}}{2\sqrt{63}} + \dfrac{5}{3}$

$$= \frac{\sqrt{4 \times 7} + \sqrt{49 \times 7}}{2 \times \sqrt{9 \times 7}} + \frac{5}{3}$$

$$= \frac{2\sqrt{7} + 7\sqrt{7}}{6\sqrt{7}} + \frac{5}{3} = \frac{9\sqrt{7}}{6\sqrt{7}} + \frac{5}{3}$$

$$= \frac{9}{6} + \frac{5}{3} = \frac{19}{6} = 3\frac{1}{6}$$

Question 4

A car dealer made a profit of 22.5 % by selling a car for GH¢58,000.00. Find, correct to **two** decimal places, the percentage profit if the car had been sold for GH¢61,200.00.

Solution

Let x be cost price

$100\% + 22.5\% = 122.5\%$

So $\quad \dfrac{122.5}{100} \times x = 58000$

$x = \dfrac{58000}{122.5} \times 100 \qquad\qquad x = 47,346.94$

$\therefore$ Cost Price $= $ GH 47,346.94

Percentage profit $= \dfrac{\text{Selling price} - \text{Cost}}{\text{Cost}} \times 100$

Percentage profit $= \dfrac{61,200 - 47,346.94}{47,346.94} \times 100$

Percentage profit $= \dfrac{13,853.06}{47,346.94} \times 100$

$\therefore$ Percentage profit $= 29.2586 = 29.26\%$

Question 5

a) A number is chosen at random from $Q = \{1, 2, 3, \ldots, 10\}$. Find the probability that the chosen number is either a prime factor of 42 or a multiple of 3.

b) If $110_x = 1020_{four}$, find the value of x.

Solution

a) $Q = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Prime factors of $42 = \{2, 3, 7\}$

So P(Prime factors of 42) $= \dfrac{3}{10}$

Multiples of $3 = \{3, 6, 9\}$

So P(Multiples of 3) $= \dfrac{3}{10}$

Prime factors of 42 and multiples of $3 = \{3\}$

P(Prime factors of 42 and multiples of 3) $= \dfrac{1}{10}$

$\therefore$ P(Prime factor of 42 **or** multiples of 3)

$= \dfrac{3}{10} + \dfrac{3}{10} - \dfrac{1}{10} = \dfrac{5}{10} = \dfrac{1}{2}$

b) $110_x = 1020_{four}$

$(1 \times x^2) + (1 \times x^1) + (0 \times x^0)$

$= (1 \times 4^3) + (0 \times 4^2) + (2 \times 4^1) + (0 \times 4^0)$

$x^2 + x + 0 = 64 + 0 + 8$

$x^2 + x = 72$

$x^2 + x - 72 = 0$

$x^2 - 8x + 9x - 72 = 0$

$x(x - 8) + 9(x - 8) = 0$

$(x - 8)(x + 9) = 0$

$\therefore x = 8$ since x cannot be negative 9.

Question 6

a) If $a = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and

$\quad r = a + \frac{1}{2}(a - b)$, find:

$\quad$ i) r;

$\quad$ ii) $|r|$.

b) Given that $a = bc$ and $n = \dfrac{mk}{ec}$,

$\quad$ i) express k in terms of a, b, e, m and n;

$\quad$ ii) find, correct to **three** significant figures, the value of k, when $a = \frac{1}{2}$, $b = -4$, $e = 3$, $m = 7$ and $n = -5$.

Solution

a) i) $r = a + \frac{1}{2}(a - b)$

$$r = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{1}{2}\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix}\right] = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

ii) $\therefore |r| = \sqrt{1^1 + 2^2} = \sqrt{5}$ units

b) i) $a = bc$, $\quad n = \dfrac{mk}{ec}$, $\quad c = \dfrac{a}{b}$

This implies $\quad n = \dfrac{mk}{\frac{ae}{b}} \qquad n = \dfrac{bmk}{ae}$

Hence $\quad k = \dfrac{aen}{bm}$

ii) $k = \dfrac{\left(\frac{1}{2}\right)(3)(5)}{(-4)(7)} = \dfrac{\frac{-15}{2}}{-28}$

$\therefore k = \dfrac{15}{2 \times 28} = 0.268$ (3s.f)

Question 7

a) Copy and complete the table of values for
$y = 5\sin x + 9\cos x$ for $0° \le x \le 150°$.

x	0°	30°	60°	90°	120°	150°
y		10.3			-0.2	

b) Using a scale of 2 cm to 30° on the x-axis
and 2 cm to 2 units on the y-axis, draw the
graph of $y = 5\sin x + 9\cos x$ for
$0° \le x \le 150°$.

c) Use the graph to solve the equations:
 i) $\quad 5\sin x + 9\cos x = 0$;
 ii) $\quad 5\sin x + 9\cos x = 2$

d) Using the graph, find, the value of y when
$x = 45°$.

Solution

a) When $x = 0°$, $y = 5\sin 0° + 9\cos 0° = 9.0$
When $x = 60°$, $y = 5\sin 60° + 9\cos 60° = 8.8$
When $x = 90°$, $y = 5\sin 90° + 9\cos 90° = 5.0$
For $x = 150°$, $y = 5\sin 150° + 9\cos 150° = -5.3$

The complete table is as follows:

x	0°	30°	60°	90°	120°	150°
y	9.0	10.3	8.8	5.0	-0.2	-5.3

b) The graph is as follows:
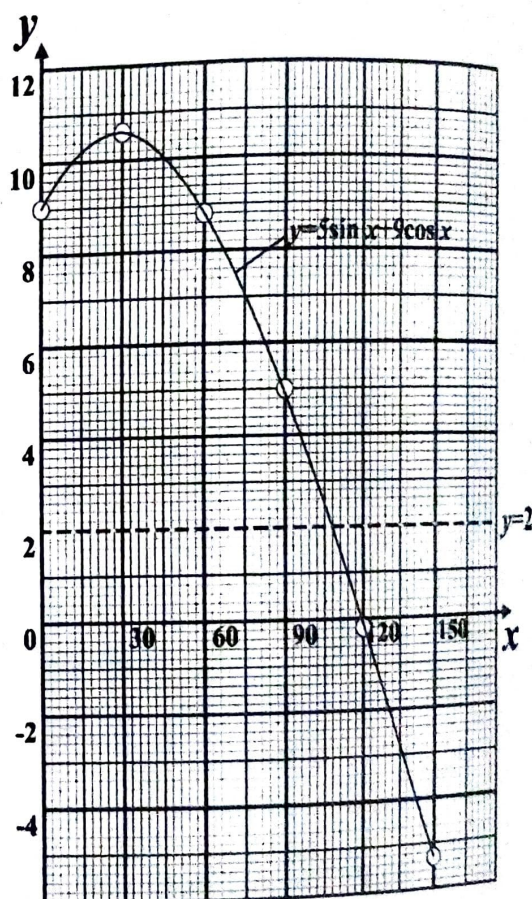
c) i) $5\sin x + 9\cos x = 0$; But $y = 5\sin x + 9\cos x$.
 This implies that $y = 0$ and so the solution
 is where the curve cuts the x-axis.
 $\therefore$ From the graph, $x = 118.5°$

ii) $5\sin x + 9\cos x = 2$ But $y = 5\sin x + 9\cos x$.

This implies that $y = 2$ and so the solution is where the curve cuts the line $y = 2$.

From the graph, $x = 108°$

d) From the graph, when $x = 45°$, $y = 9.8$

Question 8

a) Using a ruler and a pair of compasses only, construct:

i) the quadrilateral $ABCD$ such that
$|AB| = 6.5$ cm, $|BC| = 9$ cm, $|AD| = 4$ cm,
$\angle ABC = 60°$ and $\angle BAD = 120°$;

ii) the perpendicular bisectors of $\overline{BC}$ and $\overline{CD}$

b) Locate the point of intersection, T, of the two bisectors in 8(a)(ii).

c) With the point T in 8(b) as centre, draw a circle to pass through the vertices B, C and D.

d) Measure:

i) $|BT|$;   ii) $|CD|$.

Solution

a) Construct $\angle ABC = 60°$ and $\angle BAD = 120°$ as shown below:

b) T is shown on the diagram.

c) The construction is as follows:

d) i) $|BT| = 4.8$ cm

ii) $|CD| = 6$ cm

Question 9

a) Using a scale of 2 cm to 1 unit on both axes, draw on a sheet of graph paper, two perpendicular axes Ox and Oy for
$-5 \leq x \leq 5$ and $-5 \leq y \leq 5$.

b) Draw on the same graph sheet, indicating clearly all vertices and their coordinates:

i) $\triangle ABC$ with vertices $A(2, 1)$, $B(1, 4)$ and $C(-1, 2)$;

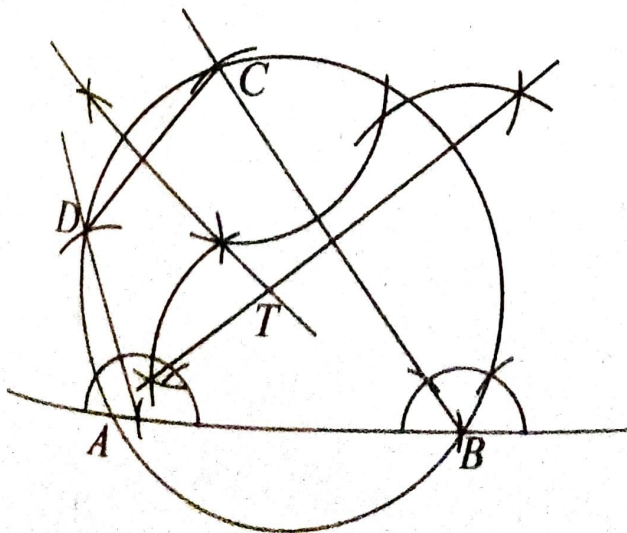ii) the image $\triangle A_1 B_1 C_1$ of $\triangle ABC$ under a reflection in the line $y = 0$, where $A \rightarrow A_1$, $B \rightarrow B_1$ and $C \rightarrow C_1$

iii) the image $\triangle A_2 B_2 C_2$ of ABC under a translation by the vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, where
$A \rightarrow A_2$, $B \rightarrow B_2$ and $C \rightarrow C_2$.

iv) the image $\triangle A_3 B_3 C_3$ of ABC under an anticlockwise rotation of $90°$ about the origin, where $A \rightarrow A_3$, $B \rightarrow B_3$ and $C \rightarrow C_3$.

c) What single transformation maps $\triangle A_1 B_1 C_1$ onto $\triangle A_3 B_3 C_3$ where $A_1 \rightarrow A_3$, $B_1 \rightarrow B_3$ and $C_1 \rightarrow C_3$.

Solution

b) i) $A(2, 1)$, $B(1, 4)$ and $C(-1, 2)$

ii) $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$ $A\begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightarrow A_1\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$B\begin{pmatrix} 1 \\ 4 \end{pmatrix} \rightarrow B_1\begin{pmatrix} 1 \\ -4 \end{pmatrix}$ $C\begin{pmatrix} -1 \\ 2 \end{pmatrix} \rightarrow C_1\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

iii) $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$A\begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} = A_2\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$B\begin{pmatrix} 1 \\ 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} = B_2\begin{pmatrix} -1 \\ 5 \end{pmatrix}$

$C\begin{pmatrix} -1 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} = C_2\begin{pmatrix} -3 \\ 3 \end{pmatrix}$
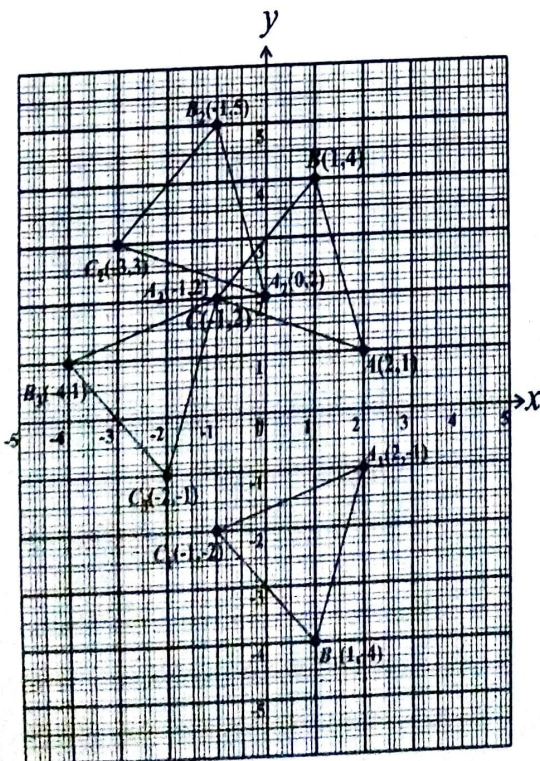
iv) $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$

$A\begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightarrow A_3\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$B\begin{pmatrix} 1 \\ 4 \end{pmatrix} \rightarrow B_3\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$C\begin{pmatrix} -1 \\ 2 \end{pmatrix} \rightarrow C_3\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

The graph is as follows:

c) $A_1\begin{pmatrix} 2 \\ -1 \end{pmatrix} \rightarrow A_3\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\quad B_1\begin{pmatrix} 1 \\ -4 \end{pmatrix} \rightarrow B_3\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$C\begin{pmatrix} -1 \\ -2 \end{pmatrix} \rightarrow C_3\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ $\quad \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ x \end{pmatrix}$

Hence the single transformation that maps $\triangle A_1B_1C_1$ onto $\triangle A_3B_3C_3$ is a reflection in the line $x = y$ or line $x - y = 0$.

Question 10

a) In a class of 50 students, 24 like football, 21 basketball and 18 cricket. 6 like football and basketball only, 3 like basketball only, 5 like all the three games and 14 did not like any of the three games.
   i) Illustrate this information on a *Venn diagram*.
   ii) Find the number of students who like;
   α) football and cricket **only**;
   β) exactly **one** of the games.

b) If $(3 - a)$, 6, $(7 - 5a)$ are consecutive terms of a Geometric Progression (G.P) with common ratio $r > 0$, find the values of a.

Solution

a) i) Let $U = \{$students in the class$\}$
      Let $x = \{$students who like football and cricket only$\}$
      The Venn diagram is as follows:
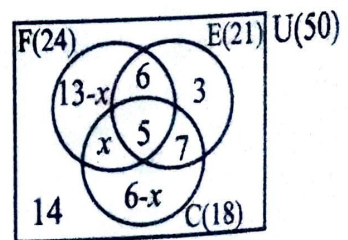
ii) α) $n(U) = 50$
      $13 - x + 6 + 5 + x + 3 + 7 + 6 - x + 14 = 50$
      $x = 54 - 50$
      $x = 4$
      ∴ 4 students like football and cricket only.

β) Students who like exactly one of the games

$= (13-x)+x+(6-x) = 9+3+2 = 14$

b) The consecutive terms of the Geometric Progression are $(3-a)$, 6, $(7-5a)$
The common ratio of the G.P is:

$\frac{6}{3-a} = \frac{7-5a}{6}$

Cross-multiplying gives:

$(3-a)(7-5a) = 6(6)$

$5a^2 - 22a - 15 = 0 \qquad (a-5)(5a+3) = 0$

$a - 5 = 0$ or $5a + 3 = 0$

$\therefore$ The values of a are 5 and $-\frac{3}{5}$.

Question 11

a) Two passenger trains, A and B, 450 km apart, start to move towards each other at the same time and meet after 2 hours. If train B, travels $\frac{8}{7}$ as fast as train A, find the speed of **each** train.

b) A solid cube of side 8 cm was melted to form a solid circular cone. The base radius of the cone is 4 cm. Calculate, correct to one decimal place, the height of the cone. [Take $\pi = \frac{22}{7}$]

Solution

a) Let speed of train $A = x$

Speed of train $B = \frac{8}{7}x$

Distance = speed × time

Distance by A corresponds to $x \times 2 = 2x$

Distance by B corresponds to $\frac{8}{7}x \times 2 = \frac{16}{7}x$

Distance coverd by A and B = 450 km

$2x + \frac{16}{7}x = 450$

Multiplying through by 7 gives:

$14x + 16x = 3150$ which gives $30x = 3150$

$\therefore x = 105$

Hence speed of train $A = 105 \text{km}^{-1}$

Now speed of train $B = \frac{8}{7}x = \frac{8}{7} \times 105$

$\therefore$ Speed of train $B = 120 \text{kmh}^{-1}$

b) Volume of cube $= 8^3 = 512 \text{ cm}^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of cone $= \frac{1}{3} \times \frac{22}{7} \times 4^2 \times h$

$\therefore$ Volume of cone $= 16.762 h$

Volume of cube = volume of cone

$512 = 16.762 h \qquad h = \frac{512}{16.762} = 30.545$

$\therefore$ Height of cone $= 30.5$ cm (1dp)

Question 12
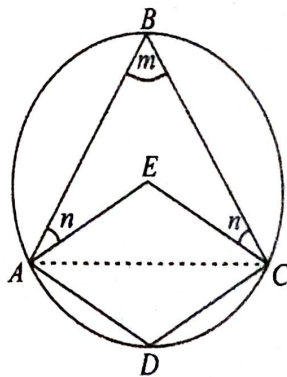
NOT DRAWN TO SCALE

a) The diagram shows a circle $ABCD$ with centre E. Quadrilateral $EADC$ is a rhombus, $\angle BAE = \angle ECB = n$ and $\angle ABC = m$.
Find:
i) m;
ii) n.

b) Find the quadratic equation whose roots are $\frac{3}{4}$ and -4

Solution

a) i) $\angle AEC = 2m$

$\angle ADC = 180 - m$

Hence $2m = 180 - m$

$3m = 180 \qquad\qquad \therefore m = 60^\circ$

ii) $\angle AEC = 2m = 2 \times 60° = 120°$

$\triangle EAC$ is isosceles with base angles EAC

and $ECA = \frac{1}{2}(180 - 120) = 30°$

$\angle EAC = \angle ECA = 30°$

From $\triangle ABC$,

$m + \angle BAD + \angle BCD = 180°$

$60 + (n + 30) + (n + 30) = 180$

$2n + 120 = 180$ $\qquad\qquad 2n = 60$

$\therefore n = 30°$

b) Method 1

Sum of roots $= \frac{3}{4} + (-4) = \frac{-13}{4}$

Product of roots $= \frac{3}{4} \times (-4) = -3$

The required quadratic equation is given by:

$x^2 - (\text{Sum of roots})\, x + \text{Product of roots} = 0$

$x^2 - \left(\frac{-13}{4}\right)x + (-3) = 0 \implies x^2 + \frac{13}{4}x - 3 = 0$

Multiplying through by 4 gives us:

$4x^2 + 13x - 12 = 0$

Method 2

Since the roots are: $\frac{3}{4}$ and -4, the required

equation is given by:

$(x - \frac{3}{4})(x + 4) = 0$

Expanding this equation gives us:

$x^2 + 4x - \frac{3}{4}x - 3 = 0$

Multiplying through by 4 gives us:

$4x^2 + 16x - 3x - 12 = 0$

$\therefore 4x^2 + 13x - 12 = 0$

Question 13

a) The fourth term of an Arithmetic Progression (A.P) is **one less than twice** the second term. If the sixth term is 7, find the first term.

b) A clerk spends $\frac{1}{5}$, $\frac{1}{3}$ and $\frac{1}{8}$ of his annual salary on rent, transport and entertainment respectively. If after all these expenses he had GH¢4,100.00 left, find how much he earns per annum.

c) Given that $f : x \rightarrow 2x^2 - 8x + 5$,

$\qquad\qquad g : x \rightarrow x - 2$;

Find: i) $f(-3)$

ii) the values of x such that $f(x) = g(x)$.

Solution

a) The nth term of an A.P is given by:

$U_n = a + (n-1)d$

where a = first term and

d = common difference.

So the fourth term of the A.P is: $U_4 = a + 3d$

The second term of the A.P is: $U_2 = a + d$

The fourth term of the A.P is **one less than twice** the second term means that:

$U_4 = (2 \times U_2) - 1$

$a + 3d = 2 \times (a + d) - 1$

$a + 3d = 2a + 2d - 1$

$-a + d = -1 \text{---------------(1)}$

The sixth term is 7 means that:

$a + 5d = 7 \text{-------------(2)}$

$(1) + (2): \ 6d = 6$ $\qquad\qquad \therefore d = 1$

Putting $d = 1$ into (1) gives: $\qquad -a + 1 = -1$

$\therefore a = 2$

Hence the first term is 2.

b) Total fraction $= \frac{1}{5} + \frac{1}{3} + \frac{1}{8} = \frac{79}{120}$

Fraction of salary left $= 1 - \frac{79}{120} = \frac{41}{120}$

Let x be annual salary;

$\frac{41}{120} \times x = 4{,}100$ $\qquad\qquad x = \frac{4{,}100 \times 120}{41}$

$\therefore x = $ GH¢ 12,000.00

Hence annual salary is GH¢ 12,000.00

c) i) $f : x \rightarrow : 2x^2 - 8x + 5$

$\quad f(-3) = 2(-3)^2 - 8(-3) + 5 = 18 + 24 + 5f$

$\quad \therefore f(-3) = 47$

ii) $f(x) = g(x)$

$2x^2 - 8x + 5 = x - 2$

$2x^2 - 8x + 5 - x + 2$

$2x^2 - 9x + 7 = 0$

$(x - 1)(2x - 7) = 0$

Hence $x = 1$ or $x = \dfrac{7}{2}$

#	2012NOV	2013	2013NOV	2014	2014NOV	2015	2015NOV	2016	2016NOV	2017
1	B	C	C	B	B	C	C	D	B	C
2	C	B	C	A	A	C	A	C	B	B
3	A	C	D	C	A	B	D	B	C	B
4	C	A	C	C	B	B	D	C	D	C
5	C	B	A	B	D	C	B	C	B	C
6	B	B	B	A	C	C	D	B	C	A
7	B		B	D	D	A	D	B	B	
8	A	C	D	B	B	B	B	A	A	
9	D	A	C	A	C	C	D	D	D	
10	D	D	D	A	B	D	A	A	B	C
11	B	C	B	C	A	C	A	C	D	C
12	C	A	B	D	D	B	C	C	C	A
13	A	B	B	D	D	B	D	B	C	C
14	D	D	C	A	B	A	B	D	C	B
15	C	A	C	B	C	C	A	A	B	A
16	B	C	B	B	A	A	C	B	C	A
17	D	C	B	D	C	B	D	A	C	A
18	A	B	B	A	D	B	C	B	D	A
19	C	C	C	A	C	C	C	B	B	B
20	B	B	B	C	C	B	C	A	A	B
21	D	C	B	B	B	B	C	C	B	B
22	D	C	B	B	D	B	A	A	B	B
23	C	A	C	B	C	C	C	C	C	A
24	B	A	C	B	B	C	B	A	A	D
25	D	D	D	A	A	D	C	A	B	B
26	A	C	D	D	A	A	B	C	B	C
27	A	C	C	C	B	B	B	D	B	C
28	B	C	C	C	A	D	B	A	A	A
29	D	D	B	A	C	C	C	B	C	A
30	C	D	B	C	A	D	A	C	A	B
31	B	A	C	C	C	A	A	B	C	C
32	A	A	B	D	A	C	B	C	C	B
33	C	D	B	B	C	A	D	D	A	C
34	B	D	A	C	C	A	A	C	C	A
35	C	D	A	C	D	D	B	D	B	C
36	C	C	C	D	C	A	A	C	D	B
37	A	D	D	A	D	D	D	B	C	D
38	A	A	C	C	D	A	D	A	D	A
39	A	C	C	A	D	B	C	B	B	B
40	D	A	B	C	A	A	A	A	D	B
41	D	A	B	B	D	D	D	A	D	D
42	A	C	C	C	D	A	B	C	B	D
43	B	C	D	C	D	A	B	A	A	A
44	D	C	C	D	D	B	B	D	D	C
45	C	C	B	B	B	A	D	B	D	C
46	D	D	B	C	A	D	B	B	D	D
47	A	D	C	D	D	B	B	D	D	D
48	C	A	B	D	A	B	A	A	D	D
49	A	A	A	C	C	A	D	D	B	C
50	D	A	C	A	D	D	A	D	D	B

615

2017NOV		2018		2018NOV		2019	
1.	A	1.	B	1.	B	1.	B
2.	A	2.	A	2.	D	2.	D
3.	D	3.	D	3.	B	3.	C
4.	B	4.	B	4.	A	4.	A
5.	A	5.	D	5.	B	5.	C
6.	C	6.	A	6.	C	6.	D
7.	A	7.	C	7.	D	7.	A
8.	D	8.	B	8.	C	8.	B
9.	D	9.	D	9.	C	9.	C
10.	C	10.	C	10.	B	10.	B
11.	D	11.	A	11.	B	11.	C
12.	D	12.	C	12.	A	12.	B
13.	C	13.	B	13.	D	13.	C
14.	B	14.	B	14.	D	14.	B
15.	C	15.	C	15.	B	15.	C
16.	D	16.	C	16.	D	16.	C
17.	D	17.	C	17.	B	17.	D
18.	A	18.	A	18.	C	18.	A
19.	C	19.	A	19.	C	19.	B
20.	B	20.	B	20.	B	20.	A
21.	D	21.	B	21.	B	21.	D
22.	D	22.	B	22.	D	22.	B
23.	A	23.	A	23.	A	23.	B
24.	A	24.	D	24.	A	24.	C
25.	D	25.	A	25.	A	25.	B
26.	B	26.	B	26.	C	26.	B
27.	D	27.	C	27.	B	27.	D
28.	C	28.	D	28.	A	28.	C
29.	D	29.	C	29.	D	29.	C
30.	A	30.	C	30.	B	30.	D
31.	A	31.	D	31.	A	31.	D
32.	D	32.	C	32.	D	32.	D
33.	D	33.	D	33.	B	33.	C
34.	B	34.	D	34.	A	34.	A
35.	B	35.	C	35.	C	35.	A
36.	C	36.	D	36.	C	36.	B
37.	B	37.	A	37.	B	37.	A
38.	A	38.	A	38.	B	38.	C
39.	C	39.	B	39.	B	39.	B
40.	B	40.	D	40.	C	40.	B
41.	B	41.	A	41.	A	41.	C
42.	B	42.	A	42.	B	42.	C
43.	D	43.	D	43.	B	43.	B
44.	C	44.	B	44.	D	44.	A
45.	D	45.	C	45.	B	45.	B
46.	B	46.	D	46.	B	46.	B
47.	C	47.	D	47.	B	47.	C
48.	D	48.	B	48.	D	48.	B
49.	D	49.	A	49.	A	49.	B
50.	D	50.	A	50.	B	50.	A